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BUCKLING AND POST-BUCKLING BEHAVIOR OF A LARGE ASPECT RATIO ORTHOTROPIC PLATE UNDER COMBINED LOADINGS.

Fred Lewis Ames

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BUCKLING AND POST-BUCKLING BEHAVIOR OF A LARGE ASPECT RATIO ORTHOTROPIC PLATE UNDER COMBINED LOADINGS

by

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(1968)

SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE

DEGREE OF OCEAN ENGINEER

AND THE DEGREE OF

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June, 1973

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by

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Lieutenant, United States Coast Guard

Submitted to the Departments of Ocean Engineering and Mechanical Engineering on 11 May 1973 in partial fullfillment of the requirements for the degrees of Ocean Engineer and Master of Science in Mechanical Engineering.

ABSTRACT

Orthotropic plate theory has been increasingly used to model plate-stiffener combinations typical of those which are used in ship hulls. The behaviour of a thin plate with large deflections is described by two nonlinear partial differential equations of equilibrium and compatibility. The orthotropic form of these equations is derived and solved for a large aspect ratio plate with long edges simply supported and short edges fixed as boundary conditions. Buckling and post-buckling regions are investigated under combined loadings of lateral pressure, inplane edge compression and edge shear. Results are presented for virtual aspect ratios 1/1.5 to 1/6 and both isotropic and orthotropic plate properties in the form of design and behaviour charts.

Thesis Supervisor: Alaa E. Mansour

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NOMENCLATURE

y = ordinate in long direction

x = ordinate in short direction

a = plate length in x-direction

b = plate breadth in y-direction

$\beta = \frac{b}{a}$ = aspect ratio

u, v, w = displacements of a point in x-, y-, and z-directions respectively

$\varepsilon_x, \varepsilon_y, \varepsilon_{xy}$ = middle-plane strains

D_x, D_y = flexural rigidity of orthotropic plate in x- or y-directions respectively

D_{xy} = effective torsional rigidity of orthotropic plate

ν_x, ν_y = Poisson's ratio of orthotropic plate in x- or y-directions respectively

E_x, E_y = modulus of elasticity in x- or y-directions respectively

G = modulus of elasticity in shear

N_x, N_y, N_{xy} = middle-plane loads per unit length

M_x, M_y = bending moment in orthotropic plate acting around a line perpendicular to x- or y-axis respectively, per unit width

M_{xy} = twisting moment in orthotropic plate

b_{mn} = nondimensional deflection coefficient

F = Airy's stress function

$J_x = \frac{1}{hE_y}$

h = plate thickness, orthotropic

$J_y = \frac{1}{hE_x}$

$2J_{xy} = \frac{1}{Gh} - \nu_x J_y - \nu_y J_x$

$2D_{xy} = D_x \nu_y + D_y \nu_x + 4C$

$C = \frac{Gh^3}{12}$

$\theta = 12\frac{a^2}{b^2}(1-\nu_x \nu_y)$

P_x, P_y = total loads in x- and y-directions respectively

$\bar{q}$ = uniform lateral load, per unit area of plate

$\bar{S}$ = constant inplane shear load, per unit length

$\rho = \frac{a}{b}\sqrt{\frac{D_y}{D_x}}$ = virtual aspect ratio of orthotropic plate

$\eta = \frac{D_{xy}}{\sqrt{D_x D_y}}$ = torsion coefficient of orthotropic plate

$\gamma = \frac{J_{xy}}{\sqrt{J_x J_y}}$

$N_x^* = \frac{\bar{N}_x a^2}{\pi^2 D_x}$ = nondimensional in-plane load x-direction

$N^*_y = \dfrac{\bar{N}_y b^2}{\pi^2 D_y}$ = nondimensional in-plane load y-direction

$Q^* = \dfrac{\bar{q} b^4}{\pi^4 h D_y}$ = nondimensional lateral load

$S^* = \dfrac{\bar{S} a b}{\pi^2 D_{xy}}$ = nondimensional edge shear load

b_e, a_e = effective width, x- or y-directions respectively

$\sigma_x, \sigma_y ; \sigma^*_x, \sigma^*_y$ = membrane stresses in x- and y-directions respectively; nondimensional

$\tau_{xy} ; \tau^*_{xy}$ = shear stress in xy plane; nondimensional

$\sigma_{bx}, \sigma_{by}; \sigma^*_{bx}, \sigma^*_{by}$ = bending stresses in x- and y-directions respectively; nondimensional

$\tau_{bxy} ; \tau^*_{bxy}$ = twisting stress in xy plane; nondimensional

$\sigma_{tx}, \sigma_{ty}, \tau_{txy}$ = total stresses

$\sigma^*_{tx}, \sigma^*_{ty}, \tau^*_{txy}$ = nondimensional total stresses

$\sigma^{*'}_{tx}, \sigma^{*'}_{ty}, \tau^{*'}_{txt}$ = nondimensional total stresses

$\sigma_{1,2} ; \sigma^*_{1,2}$ = maximum and minimum principal stresses, plane stress; nondimensional

$\sigma_{x_e} , \sigma_{y_e}$ = edge membrane stress in the x- or y-direction respectively

M_x^*, M_y^* = nondimensional bending moments per unit length

$M_{x_o}^*$, $M_{y_o}^*$ = nondimensional bending moments per unit length due to curvature in x-direction only ($M_{x_o}^*$) and y-direction only ($M_{y_o}^*$)

C_{pq}, ϕ_{pq}, A_{pq}, X_q, Y_q = coefficients

σ_o ; σ_o^* = Von Mises combined stress; nondimensional

I_x, I_y = moments of inertia of the stiffeners with effective plating in the x- and y-directions respectively

I_{px}, I_{py} = moments of inertia of the effective plating alone in the x- and y-directions respectively

S_x, S_y = spacings of the stiffeners extending in the x- and y-directions respectively

h_x, h_y = equivalent thickness of the plate and the stiffeners (diffused) in the x- and y-directions respectively

h_p = thickness of plate alone

1. Introduction

Of major concern to the naval engineer is the reaction of ship bottom plates to a combination of inservice loadings. In general, the plate-stiffener combinations will be subjected to inplane compression or tension edge loads due to ship hogging or sagging, lateral hydrostatic loads and inplane edge shear loads. With increasing compression and shear loads, a critical state will be reached where buckling will occur. Plate deflection will exceed plate thickness, however, the plate may still carry considerable loads. High slenderness ratio (b/h) plates are encountered in modern longitudinally framed ship designs. The yielding load may be considerably higher than the buckling load and investigation of the plate-stiffener residual strength in the post-buckling region is of great importance.

Orthotropic plate theory has been increasingly applied to ship structures. With this analysis, the plate-stiffener combination is modeled as an equivalent flat plate with elastic properties that are different in two perpendicular directions. Schade [12, 13] and Mansour [9, 10]*, among others, have worked with this concept.

The small deflection theory is useful only in the pre-buckling region where deflections of a plate are small in

---

*Numbers in brackets designate references in Chapter 8.

comparison with its thickness. The assumption of no deformation in the middle plane of the plate is made in this case. If deflections are not small, $\frac{w}{h} > \frac{1}{2}$, the strain in the middle plane is no longer negligible and must be considered. Von Karman's equations describe the behavior of an isotropic plate under combined lateral and inplane loads. The nonlinear equations describe the behavior of the plate in both small and large deflection regions.

Considerable application has been made of von Karman's equations. Levy [4, 6] investigated square and rectangular plates simply supported and subjected to both inplane edge loading and normal pressure. Coan [3] included the effects of small initial curvature for this case. Levy [5], using two fixed and two simply supported ends, solved the equations with edge loading only for an aspect ratio of 4, and studied pure shear for the simply supported square plate [7, 8]. The square plate with variations of the boundary conditions, subjected to edge loading only, was studied by Yamaki [17]. Payer [11] applied von Karman's equations to deep web frames and included uniform edge shear in addition to compressive edge stress and normal pressure. Shultz [15] investigated wide plates having aspect ratios of $\beta = 1.5$ to 8 for conditions of a transversely framed ship. All of these analyses apply only to isotropic plates. Mansour [9, 10] extended von Karman's equations to consider slightly rectangular orthotropic plates under various boundary, loading and initial deflection conditions.

A large aspect ratio orthotropic plate with the short edges fixed and the long edges simply supported is considered in this paper. The plate-stiffener combination is subjected to loads of inplane edge compression, uniform edge shear and uniform pressure normal to the plate. Figure 1 illustrates the loadings and coordinate system used.

The orthotropic form of von Karman's equations are solved similar to the theoretical analysis of Mansour [9, 10]. The IBM System 370 model 165 is employed to produce the numerical results. "Design" charts of deflection, effective width and bending moment are given for orthotropic plate virtual aspect ratios of 1/1.5 to 1/6 with inplane edge compressive and lateral loadings. In addition, "behaviour" plots of plate centerline deflection and total principal stresses are given for virtual aspect ratios of 1/1.5 and 1/1.25 for various combinations of inplane edge compression, edge shear and lateral loadings.

2. Formulation of the Problem

2.1 Orthotropic Material Properties

An orthotropic material has different elastic properties in two perpendicular directions. For plane stress in the xy plane, the stress-strain relations are

$$\sigma_x = \frac{E_x}{1-\nu_x\nu_y}(\varepsilon_x + \nu_y\varepsilon_y)$$

$$\sigma_y = \frac{E_y}{1-\nu_x\nu_y}(\varepsilon_y + \nu_x\varepsilon_x) \tag{1}$$

$$\tau_{xy} = \gamma_{xy}G$$

from energy symetry

$$E_x\nu_y = E_y\nu_x$$

which gives 4 independent elastic constants.

2.2 The Rectangular Orthotropic Plate with Large Deflection (Figure 1)

For thin plates with large deflection, $\frac{w}{h} > \frac{1}{2}$, a satisfactory approximate theory makes the following assumptions:

1. Points initially on the normal to the middle plane of the plate remain on the normal after bending. (Disregard of shear deformation.)

2. The normal stresses in the direction transverse to the plate can be disregarded. (Plane stress where $\sigma_z = 0$.)

3. Hooke's Law relating stress-strain applies.

For the large deflection theory, deformation in the middle plane of the plate must be considered. These strain components therefore include the effect of deflection and are approximately

$$\begin{aligned} \varepsilon_x &\approx \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\ \varepsilon_y &\approx \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \\ \gamma_{xy} &\approx \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y} \end{aligned} \tag{2}$$

Equilibrium of forces and moments on an element of a plate produces the equilibrium equations

$$\begin{aligned} \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} &= 0 \\ \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} &= 0 \end{aligned} \tag{3}$$

and

$$\frac{\partial^2 M_x}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2}$$

$$= -\left(\bar{q} + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2}\right) \tag{4}$$

Introducing the familiar Airy's stress function F which satisfies equations (3)

$$N_x = \frac{\partial^2 F}{\partial y^2} \; ; \; N_y = \frac{\partial^2 F}{\partial x^2} \; ; \; N_{xy} = -\frac{\partial^2 F}{\partial x \partial y} \qquad (5)$$

From substitution of large deflection strains (2) into the stress-strain relations (1) and using the definition of bending and twisting moments, the moment-curvature relations for an orthotropic material are derived

$$\begin{aligned} M_x &= -D_x\left(\frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2}\right) \\ M_y &= -D_y\left(\frac{\partial^2 w}{\partial y^2} + \nu_x \frac{\partial^2 w}{\partial x^2}\right) \qquad (6) \\ M_{xy} &= 2C \frac{\partial^2 w}{\partial x \partial y} \end{aligned}$$

where the rigidity coefficients are defined as

$$D_x \equiv \frac{E_x h^3}{12(1-\nu_x \nu_y)}$$

$$D_y \equiv \frac{E_y h^3}{12(1-\nu_x \nu_y)}$$

and

$$C \equiv \frac{Gh^3}{12}$$

Substitution of equations (6) and equations (5) into equation (4), the "equilibrium" equation is obtained as

$$D_x\frac{\partial^4 w}{\partial x^4} + 2D_{xy}\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y\frac{\partial^4 w}{\partial y^4}$$

$$= \bar{q} + \frac{\partial^2 F}{\partial y^2} \cdot \frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{\partial x \partial y} \cdot \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} \quad (7)$$

where

$$2D_{xy} = D_x \nu_y + D_y \nu_x + 4C$$

To obtain the "compatibility" equation, the strain equations (2) are first differentiated and combined to eliminate displacements u and v

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} \quad (8)$$

Differentiation of equations (1), substitution into equation (8) along with (5) and using the equilibrium equations (3), the "compatibility" equation is obtained as

$$J_x\frac{\partial^4 F}{\partial x^4} + 2J_{xy}\frac{\partial^4 F}{\partial x^2 \partial y^2} + J_y\frac{\partial^4 F}{\partial y^4}$$

$$= \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} \quad (9)$$

where

$$J_x = \frac{1}{E_y h} \quad ; \quad J_y = \frac{1}{E_x h}$$

and

$$2J_{xy} = \frac{1}{Gh} - \nu_x J_y - \nu_y J_x$$

Equations (7) and (9) are fourth-order non-linear partial-differential equations that describe both small and large deflections of an orthotropic plate. This includes the buckling and post-buckling behavior of the stiffened plate. Substitution of the isotropic material properties

$$E_x = E_y = E \; ; \; \nu_x = \nu_y = \nu \; ; \quad G = \frac{E}{2(1+\nu)}$$

result in the familiar von Karman's equations.

Solution of these equations with 16 boundary conditions results in the two functions F and w. The boundary conditions are eight support conditions and eight edge load/displacement conditions.

2.3 Bending and Membrane Stress

The membrane stresses are determined from equations (5) as

$$\sigma_x = \frac{N_x}{h} = \frac{1}{h} \frac{\partial^2 F}{\partial y^2}$$

$$\sigma_y = \frac{N_y}{h} = \frac{1}{h} \frac{\partial^2 F}{\partial x^2} \tag{10}$$

$$\tau_{xy} = \frac{N_{xy}}{h} = -\frac{1}{h} \frac{\partial^2 F}{\partial x \partial y}$$

Noting that the maximum normal stress acts on those sections parallel to the xz or yz planes, and using equations (1), (2) and (6) the bending and shear stresses are obtained as

$$\sigma'_{bx} = z\frac{E_x}{(1-\nu_x\nu_y)}\frac{M_x}{D_x}$$

$$\sigma'_{by} = z\frac{E_y}{(1-\nu_x\nu_y)}\frac{M_y}{D_y}$$

$$\tau'_{bxy} = -zG\frac{M_{xy}}{C}$$

The maximum bending and shear stresses occuring at $z = \pm\frac{h}{2}$ gives

$$\sigma_{bx} = \pm 6\frac{M_x}{h^2}\ ;\quad \sigma_{by} = \pm 6\frac{M_y}{h^2}\ ;\quad \tau_{bxy} = \mp 6\frac{M_{xy}}{h^2} \qquad (11)$$

where M_x and M_y and M_{xy} are given in equations (6). The total stresses are the sums

$$\begin{aligned} \sigma_{tx} &= \sigma_x \pm \sigma_{bx} \\ \sigma_{ty} &= \sigma_y \pm \sigma_{by} \\ \tau_{txy} &= \tau_{xy} \mp \tau_{bxy} \end{aligned} \qquad (12)$$

2.4 Load Boundary Conditions

The following most general conditions state that the edges are subjected to an inplane average compressive load per unit length in the y-direction and x-direction of magnitudes $\bar{N}_y$ and $\bar{N}_x$ respectively. Additionally, all edges are subjected to a constant inplane shear load per unit length of magnitude $\bar{S}$. Referring to Figure 1,

at $x = \pm\frac{a}{2}$

$$N_{xy} = \bar{S} \ , \ \frac{\partial^2 F}{\partial x \partial y} = -\bar{S}$$

Inplane load resultant P_x is (13a)

$$P_x = \int_{-b/2}^{b/2} \frac{\partial^2 F}{\partial y^2} \, dy = -\bar{N}_x b$$

at $y = \pm\frac{b}{2}$

$$N_{yx} = \bar{S} \ , \ \frac{\partial^2 F}{\partial x \partial y} = -\bar{S}$$

Inplane load resultant P_y is (13b)

$$P_y = \int_{-a/2}^{a/2} \frac{\partial^2 F}{\partial x^2} \, dx = -\bar{N}_y a$$

2.5 Support Conditions

Edges simply supported at $x = \pm\frac{a}{2}$

Deflection equals zero: $w = 0$

External moments equal zero: (14a)

$$\frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2} = 0$$

Edges clamped at $y = \pm\frac{b}{2}$

Deflection equals zero: $w = 0$

Slope equals zero: $\frac{\partial w}{\partial y} = 0$ (14b)

3. Theoretical Analysis

3.1 Analysis Procedure

The solution method used is identical to that of Mansour [9, 10] which is an extension of Levy [4], Coan [3] and Yamaki [17]. Briefly, the outline of the procedure is:

1. Express the deflection of the plate satisfying the support boundary conditions, in a double trigonometric series choosing only a finite number of terms. w will be a function of unknown nondimensional coefficients b_{mn}.

2. Substitution of this expression into the "compatibility" equation (9) results in a fourth order partial-differential equation with the Airy stress function F and quadratic functions of the coefficient b_{mn} expressed as coefficients C_{pq}.

3. A solution for the stress function is assumed which satisfies equation (9) and the load boundary conditions. F will be expressed as functions of unknown coefficients ϕ_{pq} which in turn are functions of coefficients C_{pq}.

4. To determine the unknown coefficients b_{mn}, Galerkin's Method is applied to the "equilibrium" equation (7)

with substitution for w and F by their appropriate expressions. Using the orthogonality properties of the trigonometric functions, a set of simultaneous, non-linear algebraic equations involving cubic products of the coefficients b_{mn} results.

3.2 Solution

The deflection of the plate surface can be expressed in the form

$$w = h \sum_m \sum_n b_{mn} \, f_m(x) \, g_n(y) \tag{15}$$

Satisfying the boundary conditions (14), the deflection terms take the form

$$f_m(x) = \cos\frac{m\pi}{a}x \quad ; \quad m = 1, 3, 5....$$

$$g_n(y) = (-1)^{n+1} + \cos\frac{2n\pi}{b}y \quad ; \; n = 1, 2, 3....$$

Differentiation of expression (15) and substitution into the "compatibility" equation (9)

$$\begin{aligned} J_x\frac{\partial^4 F}{\partial x^4} + 2J_{xy}\frac{\partial^4 F}{\partial x^2 \partial y^2} + J_y\frac{\partial^4 F}{\partial y^4} \\ = h^2\{\sum_m \sum_n b_{mn}\left(\frac{m\pi}{a}\right)\left(\frac{2n\pi}{b}\right)\sin\frac{m\pi}{a}x \, \sin\frac{2n\pi}{b}y\}^2 \\ - h^2\{\sum_m \sum_n b_{mn}[(-1)^{n+1} + \cos\frac{2n\pi}{b}y]\left(\frac{m\pi}{a}\right)^2\cos\frac{m\pi}{a}x\} \\ \cdot\{\sum_m \sum_n b_{mn}\left(\frac{2n\pi}{b}\right)^2 \cos\frac{m\pi}{a}x \, \cos\frac{2n\pi}{b}y\} \end{aligned}$$

$$= \frac{h^2\pi^4}{a^2b^2} \{\sum_m \sum_n \sum_i \sum_j 4b_{mn} b_{ij} mnij \sin\frac{m\pi}{a}x \sin\frac{2n\pi}{b}y$$

$$\cdot \sin\frac{i\pi}{a}x \sin\frac{2j\pi}{b}y - \sum_m \sum_n \sum_i \sum_j 4b_{mn} b_{ij} m^2 j^2 \cos\frac{m\pi}{a}x$$

$$\cdot [(-1)^{n+1} + \cos\frac{2n\pi}{b}y] \cos\frac{i\pi}{a}x \cos\frac{2j\pi}{b}y\} \tag{16}$$

$m, i = 1, 3, 5 \ldots.$

$n, j = 1, 2, 3 \ldots.$

which can be expressed in the form

$$J_x \frac{\partial^4 F}{\partial x^4} + 2J_{xy} \frac{\partial^4 F}{\partial x^2 \partial y^2} + J_y \frac{\partial^4 F}{\partial y^4}$$

$$= \frac{h^2\pi^4}{a^2b^2} \sum_p \sum_q C_{pq} \cos\frac{2p\pi}{a}x \cos\frac{2q\pi}{b}y \tag{17}$$

$p, q = 0, 1, 2 \ldots.$

where C_{pq} are quadratic functions of the nondimensional coefficients b_{mn}.

A particular solution for (17) is assumed in the form

$$F_p = h^2 \sum_p \sum_q \phi_{pq} \cos\frac{2p\pi}{a}x \cos\frac{2q\pi}{b}y \tag{18}$$

substitution of (18) into equation (17) defines coefficient ϕ_{pq} as

$$\phi_{pq} = \frac{\beta^2 C_{pq}}{16(J_x p^4 \beta^4 + 2J_{xy} p^2 q^2 \beta^2 + J_y q^4)} \qquad (19)$$

where

$$\beta = \frac{b}{a}$$

The stress function F to satisfy the load boundary conditions (13) and equation (9) is

$$F = -\frac{\bar{N}_y}{2}x^2 - \frac{\bar{N}_x}{2}y^2 - \bar{S}xy + F_p \qquad (20)$$

The method of B.G. Galerkin is applied to the "equilibrium" equation (7) to determine the unknown coefficients b_{mn}. Galerkin's condition requires that the following equation be satisfied by all functions $f_r(x)\ g_s(y)$

$$\int_0^{a/2} \int_0^{b/2} [D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4}$$

$$- \bar{q} - \frac{\partial^2 F}{\partial y^2} \cdot \frac{\partial^2 w}{\partial x^2} + 2\frac{\partial^2 F}{\partial x \partial y} \cdot \frac{\partial^2 w}{\partial x \partial y} - \frac{\partial^2 F}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2}] \qquad (21)$$

$$\cdot\ f_r(x)\ g_s(y)\ dxdy = 0$$

where $f_r(x)\ g_s(y)$ is given be equation (15). Equation (21) may also be obtained by applying the principle of virtual work.

Substitution of w and F, expressions (15) and (20) respectively, into equation (21) and using the orthogonality

properties of the trigonometric functions, there results

$$\sum_r \sum_s [(2s)^2 \frac{\rho^4}{\theta J_y} (\frac{\bar{N}_y b^2}{\pi^2 D_y}) + r^2 \frac{1}{\theta J_y} (\frac{\bar{N}_x a^2}{\pi^2 D_x})]\, b_{rs}$$

$$- \frac{1}{\theta J_y} \sum_n \sum_r \sum_s b_{rn} \{2(-1)^{n+s} r^4 + [r^4 + 2\rho^2 \eta (2s)^2 r^2 + \rho^4 (2s)^4]\, \delta_{ns}$$

$$- 2 \cdot r^2 (-1)^{n+s} (\frac{\bar{N}_x a^2}{\pi^2 D_x})\} + \sum_m \sum_r \sum_n^{\text{odd}} \sum_s 4m (\frac{4}{\pi})^2 \Delta_{mr} (-1)^{s+1} \frac{\eta \rho^2}{\theta J_y} (\frac{\bar{S}ab}{\pi^2 D_{xy}})\, b_{mn}$$

$$+ \sum_m \sum_r \sum_{n+s}^{\text{odd}} 2nm (\frac{4}{\pi})^2 \Delta_{mr} (\frac{1}{n+s} + \frac{1}{n-s}) \frac{\eta \rho^2}{\theta J_y} (\frac{\bar{S}ab}{\pi^2 D_{xy}})\, b_{mn}$$

$$+ \sum_m \sum_n \sum_r \sum_s b_{mn} \{2(-1)^{s+1} n^2 r^2 [\phi_{\frac{m+r}{2},n} + \phi_{\frac{m-r}{2},n} + \phi_{\frac{r-m}{2},n}]$$

$$+ 2(-1)^{n+1} m^2 s^2 [\phi_{\frac{m+r}{2},s} + \phi_{\frac{m-r}{2},s} + \phi_{\frac{r-m}{2},s}]$$

$$+ (ms-nr)^2 [\phi_{\frac{m+r}{2},n+s} + \phi_{\frac{m-r}{2},n-s} + \phi_{\frac{m-r}{2},s-n} + \phi_{\frac{r-m}{2},n-s} + \phi_{\frac{r-m}{2},s-n}]$$

$$+ (ms+nr)^2 [\phi_{\frac{m+r}{2},n-s} + \phi_{\frac{m+r}{2},s-n} + \phi_{\frac{m-r}{2},n+s} + \phi_{\frac{r-m}{2},n+s}]\}$$

$$= \sum_r \sum_s (-1)^{s+\frac{3r+1}{2}} \frac{\rho^4}{\theta J_y} \frac{8}{r\pi} (\frac{\bar{q} b^4}{\pi^4 h D_y}) \qquad (22)$$

$m, r = 1, 3, 5....$
$n, s = 1, 2, 3....$

where δ_{ij} is the Kronecker delta, $\phi_{i,j} = 0$ if $i < 0$ or $j < 0$, $\frac{1}{n-s} = 0$ if $n = s$ and

$$\Delta_{mr} = \frac{1}{m+r} \quad \text{if} \quad \frac{m+r}{2} \quad \text{odd}$$

or

$$\Delta_{mr} = \frac{1}{m-r} \quad \text{if} \quad \frac{m-r}{2} \quad \text{odd}$$

Equation (21) is identical, except for the addition of the two shear and two $\bar{N}_x$ load terms, to that of Mansour [9] case (i) and, if isotropic material properties are substituted, reduces to Yamaki [17] case IIb for zero initial curvature.

Substitution for coefficients ϕ_{pq} in equation (22) results in a set of simultaneous, non-linear algebraic equations involving cubic products of the coefficients b_{mn}. Solution of these equations gives the coefficients b_{mn} and hence defines the functions w and F.

3.3 Bending and Membrane Stresses

The membrane stresses are now determined from equations (10), (18) and (20). These are

$$\sigma_x = -\frac{\bar{N}_x}{h} - h \sum_p \sum_q \phi_{pq} \left(\frac{2q\pi}{b}\right)^2 \cos\frac{2p\pi}{a}x \cos\frac{2q\pi}{b}y$$

$$\sigma_y = -\frac{\bar{N}_y}{h} - h \sum_p \sum_q \phi_{pq} \left(\frac{2p\pi}{a}\right)^2 \cos\frac{2p\pi}{a}x \cos\frac{2q\pi}{b}y$$

$$\tau_{xy} = -\frac{\bar{S}}{h} - h \sum_p \sum_q \phi_{pq} \left(\frac{2p\pi}{a}\right) \left(\frac{2q\pi}{b}\right) \sin\frac{2p\pi}{a}x \sin\frac{2q\pi}{b}y \qquad (23)$$

The membrane stresses will be represented in the most general form, the nondimensional form used by Mansour [9]. Using equation (19)

$$\sigma_x^* = \frac{\sigma_x}{\left(\frac{\pi^2 h}{a^2 J_y}\right)} = -\frac{N_x^*}{12(1-\nu_x\nu_y)} - \sum_p \sum_q \frac{C_{pq} q^2 \rho^4}{4(p^4 + 2\gamma\rho^2 p^2 q^2 + \rho^4 q^4)} \cos\frac{2p\pi}{a}x \cos\frac{2q\pi}{b}y \tag{24}$$

$$\sigma_y^* = \frac{\sigma_y}{\left(\frac{\pi^2 h}{b^2 J_x}\right)} = -\frac{N_y^*}{12(1-\nu_x\nu_y)} - \sum_p \sum_q \frac{C_{pq} p^2}{4(p^4 + 2\gamma\rho^2 p^2 q^2 + \rho^4 q^4)} \cos\frac{2p\pi}{a}x \cos\frac{2q\pi}{b}y \tag{25}$$

and

$$\tau_{xy}^* = \frac{\tau_{xy}}{\left[\frac{\pi^2 h}{abJ_{xy}}\right]} = -\frac{S^*\eta\gamma}{12(1-\nu_x\nu_y)} - \sum_p \sum_q \frac{C_{pq} p\cdot q}{4\left(\frac{p^4}{\gamma\rho^2} + 2p^2q^2 + \frac{\rho^2}{\gamma}q^4\right)} \sin\frac{2p\pi}{a}x \sin\frac{2q\pi}{b}y$$

$$p,\ q = 0,\ 1,\ 2\ldots.$$

The bending and twisting stresses from equation (11) are

$$\sigma_{bx} = \pm \frac{6}{h^2} M_x = \pm \frac{6D_x}{h^2} \left[- \frac{\partial^2 w}{\partial x^2} - \nu_y \frac{\partial^2 w}{\partial y^2}\right] \qquad (26)$$

$$\sigma_{by} = \pm \frac{6}{h^2} M_y = \pm \frac{6D_y}{h^2} \left[- \frac{\partial^2 w}{\partial y^2} - \nu_x \frac{\partial^2 w}{\partial x^2}\right]$$

$$\tau_{bxy} = \mp \frac{6}{h^2} M_{xy} = \mp \frac{12C}{h^2} \frac{\partial^2 w}{\partial x \partial y}$$

The normal bending stresses may be expressed as functions of nondimensional bending moments M^*_x and M^*_y where

$$M^*_x = \frac{M_x}{\left[\frac{h\sqrt{D_x D_y}}{b^2}\right]} = M^*_{x_o} + \nu_y \sqrt{\frac{D_x}{D_y}}\, M^*_{y_o}$$

$$(27)$$

$$M^*_y = \frac{M_y}{\left[\frac{hD_y}{b^2}\right]} = M^*_{y_o} + \nu_x \sqrt{\frac{D_y}{D_x}}\, M^*_{x_o}$$

and

$$M^*_{x_o} = \frac{M_{x_o}}{\left[\frac{h\sqrt{D_x D_y}}{b^2}\right]} = - D_x \frac{\partial^2 w}{\partial x^2} \frac{b^2}{h\sqrt{D_x D_y}}$$

$$(28)$$

$$M^*_{y_o} = \frac{M_{y_o}}{\left[\frac{hD_y}{b^2}\right]} = - D_y \frac{\partial^2 w}{\partial y^2} \frac{b^2}{hD_y}$$

The bending stresses as functions of the nondimensional bending moments are

$$\sigma_{bx} = \pm \frac{6M^*_x}{h^2} \cdot \frac{h\sqrt{D_x D_y}}{b^2}$$

$$\sigma_{by} = \pm \frac{6M^*_y}{h^2} \cdot \frac{hD_y}{b^2} \tag{29}$$

The nondimensional moment due to curvature in the x-direction only ($M^*_{x_o}$) and that due to curvature in the y-direction only ($M^*_{y_o}$) are determined from (15) and (28)

$$M^*_{x_o} = \sum_m \sum_n \frac{\pi^2 m^2}{\rho^2} b_{mn} \cos\frac{m\pi}{a}x \; [(-1)^{n+1} + \cos\frac{2n\pi}{b}y]$$

$$M^*_{y_o} = \sum_m \sum_n 4\pi^2 n^2 b_{mn} \cos\frac{m\pi}{a}x \; \cos\frac{2n\pi}{b}y \tag{30}$$

$$m = 1,\ 3,\ 5 \ldots.$$

$$n = 1,\ 2,\ 3 \ldots.$$

The bending stresses may be expressed as nondimensional bending stresses in a form compatible to equations (24) and (25). Using equations (15) and (26), the nondimensional bending and twisting stresses are

$$\sigma^*_{bx} = \frac{\sigma_{bx}}{[\frac{\pi^2 h}{a^2 J_y}]} = \pm \frac{1}{2(1-\nu_x \nu_y)} \sum_m \sum_n b_{mn} \cos\frac{m\pi}{a}x$$

$$\cdot [m^2 (-1)^{n+1} + (m^2 + \frac{4n^2 \nu_y}{\beta^2}) \; \cos\frac{2n\pi}{b}y] \tag{31}$$

$$\sigma^*_{by} = \frac{\sigma_{by}}{\left[\frac{\pi^2 h}{b^2 J_x}\right]} = \pm \frac{1}{2(1-\nu_x \nu_y)} \sum_m \sum_n b_{mn} \cos\frac{m\pi}{a}x \tag{32}$$

$$\cdot [m^2\beta^2\nu_x(-1)^{n+1} + (4n^2 + m^2\beta^2\nu_x)\cos\frac{2n\pi}{b}y]$$

and

$$\tau^*_{bxy} = \frac{\tau_{bxy}}{\left[\frac{\pi^2 h}{abJ_{xy}}\right]} = \mp \frac{\gamma\eta}{(1-\nu_x\nu_y)} \left[1 - \frac{\nu_y}{2\eta\rho^2\beta^2} - \frac{\nu_x\rho^2\beta^2}{2\eta}\right]$$

$$\cdot \sum_m \sum_n b_{mn} nm \sin\frac{2n\pi}{b}y \sin\frac{m\pi}{a}x$$

$$m = 1, 3, 5....$$

$$n = 1, 2, 3....$$

The nondimensional total stresses are therefore, from (12)

$$\sigma^*_{tx} = \sigma^*_x \pm \sigma^*_{bx}$$

$$\sigma^*_{ty} = \sigma^*_y \pm \sigma^*_{by} \tag{33}$$

$$\tau^*_{txy} = \tau^*_{xy} \mp \tau^*_{bxy}$$

3.4 Effective Width

The effective width of a rectangular plate which has buckled is defined as that width of a uniformly stressed phantom plate of the same thickness stressed to the same maximum stress and sustaining the same total force as the real plate. The effective widths a_e and b_e, in the y and x

directions respectively are hence defined as

$$a_e\sigma_{ye} = a\,\frac{\bar{N}_y}{h} \tag{34a}$$

$$b_e\sigma_{xe} = b\,\frac{\bar{N}_x}{h} \tag{34b}$$

where σ_{ye} is the edge membrane stress in the y-direction (σ_y at $x = \pm\frac{a}{2}$) and σ_{xe} is the edge membrane stress in the x-direction (σ_x at $y = \pm\frac{b}{2}$). Substitution of (25) evaluated at $x = \pm\frac{a}{2}$ into (34a) gives

$$\frac{a_e}{a} = \frac{N_y^*}{N_y^* + 3(1-\nu_x\nu_y)\sum\limits_p\sum\limits_q \dfrac{p^2(-1)^p C_{pq}}{(p^4 + 2\gamma\rho^2p^2q^2 + \rho^4q^4)}\cos\dfrac{2q\pi}{b}y} \tag{35a}$$

$$p,\ q = 0,\ 1,\ 2....$$

or

$$\frac{a_e}{a} = \frac{N_y^*}{N_y^* + 3(1-\nu_x\nu_y)\left[\sum\limits_p \dfrac{(-1)^p}{p^2}C_{p,o} + \sum\limits_p\sum\limits_{q=1}^{\infty} \dfrac{p^2(-1)^p C_{pq}}{(p^4 + 2\gamma\rho^2p^2q^2 + \rho^4q^4)}\right]\cos\dfrac{2q\pi}{b}y}$$

substitution of (24) evaluated at $y = \pm\frac{b}{2}$ into (34b) gives

$$\frac{b_e}{b} = \frac{N_x^*}{N_x^* + 3(1-\nu_x\nu_y)\sum\limits_p\sum\limits_q \dfrac{q^2(-1)^q\rho^4 C_{pq}}{(p^4 + 2\gamma\rho^2p^2q^2 + \rho^4q^4)}\cos\dfrac{2p\pi}{a}x} \tag{35b}$$

$$p,\ q = 0,\ 1,\ 2....$$

or

$$\frac{b_e}{b} = \frac{N_x^*}{N_x^* + 3(1-\nu_x\nu_y)\left[\sum_q \frac{(-1)^q}{q^2} C_{o,q} + \sum_{p=1}^{\infty} \sum_q \frac{q^2(-1)^q \rho^4 C_{pq}}{(p^4 + 2\gamma\rho^2 p^2 q^2 + \rho^4 q^4)}\right] \cos\frac{2p\pi}{a} x}$$

Clearly, the effective widths a_e and b_e are not constant over the y and x-directions respectively. Neglecting the small amount of change due to the periodic terms in y or x, taking only the average values into account [15], the effective widths take the form

$$\frac{a_e}{a} = \frac{N_y^*}{N_y^* + 3(1-\nu_x\nu_y)\sum_p \frac{(-1)^p}{p^2} C_{p,o}} \qquad (36a)$$

$$p = 0,\ 1,\ 2....$$

and

$$\frac{b_e}{b} = \frac{N_x^*}{N_x^* + 3(1-\nu_x\nu_y)\sum_q \frac{(-1)^q}{q^2} C_{o,q}} \qquad (36b)$$

$$q = 0,\ 1,\ 2....$$

3.5 Principal Stresses

For the state of plane stress, the maximum and minimum principal stresses, σ_1 and σ_2, are given by

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Substituting the nondimensional total stresses, equations (33), the nondimensional principal total stresses are

$$\sigma^*_{1,2} = \frac{\sigma^{*\prime}_{tx} + \sigma^{*\prime}_{ty}}{2} \pm \sqrt{\left(\frac{\sigma^{*\prime}_{tx} - \sigma^{*\prime}_{ty}}{2}\right)^2 + \tau^{*\prime\,2}_{txy}} \qquad (37)$$

where

$$\sigma^*_{1,2} = \frac{\sigma_{1,2}}{\left[\frac{\pi^2 h}{b^2 J_x}\right]}$$

and

$$\sigma^{*\prime}_{tx} = \frac{\sigma^*_{tx}}{\rho^4 \beta^2}$$

$$\sigma^{*\prime}_{ty} = \sigma^*_{ty}$$

$$\tau^{*\prime}_{txy} = \frac{\tau^*_{txy}}{\gamma \rho^2 \beta}$$

4. Numerical Solution

4.1 Analytic Solution

Shultz [15] found that eight deflection terms were sufficient to describe a simply supported plate up to an aspect ratio of 8. In this solution eight deflection terms in the y-direction and one in the x-direction were assumed. From equation (15) with $m = 1$ and $n = 1,2,3\ldots8$, the deflection of the plate takes the form

$$w = h\cos\frac{\pi}{a}x\,[b_{11}(1+\cos\frac{2\pi}{b}y) + b_{12}(-1+\cos\frac{4\pi}{b}y)$$

$$+ b_{13}(1+\cos\frac{6\pi}{b}y) + b_{14}(-1+\cos\frac{8\pi}{b}y) + b_{15}(1+\cos\frac{10\pi}{b}y)$$

$$+ b_{16}(-1+\cos\frac{12\pi}{b}y) + b_{17}(1+\cos\frac{14\pi}{b}y) + b_{18}(-1+\cos\frac{16\pi}{b}y)$$

Similarly, the right hand side of equation (16) is expanded for $m, i = 1$; $n, j = 1,2,3\ldots8$. The coefficients C_{pq} are determined by collecting terms on the right hand side of equation (16) and matching the coefficients to the series on the right hand side of equation (17) for $p = 0, 1$; $q = 0, 1, 2\ldots16$. The coefficients X_q and Y_q are listed in Table 1 where the coefficients C_{pq} are given by

$$C_{pq} = Y_q + (-1)^p X_q \qquad (38)$$

$$p = 0, 1$$

$$q = 0, 1, 2\ldots16$$

Substitution of coefficients C_{pq} into equation (22) using equation (19) and with $m, r = 1$; $n, s = 1,2,3....8$ produces the following equation after considerable simplification

$$
\begin{aligned}
&\sum_s (2s)^2 N_y^* b_{1,s} + \sum_s \frac{N_x^*}{\rho^4} b_{1,s} - \frac{1}{\rho^4} \sum_n \sum_s b_{1,n} \{ 2(-1)^{n+s} \\
&+ [1+2\rho^2\eta(2s)^2 + \rho^4(2s)^4]\delta_{ns} \} + \frac{2}{\rho^4} \sum_n \sum_s b_{1,n} (-1)^{n+s} N_x^* \\
&+ 2\left(\frac{4}{\pi}\right)^2 \frac{\eta}{\rho^2} \sum_n^{odd} \sum_s b_{1,n} (-1)^{s+1} S^* + \left(\frac{4}{\pi}\right)^2 \frac{\eta}{\rho^2} \sum_{n+s}^{odd} b_{1,n}\, n \left(\frac{1}{n+s} + \frac{1}{n-s}\right) S^* \\
&+ \sum_n \sum_s b_{1,n} \{ 2(-1)^{s+1} n^2 [A_{1,n}C_{1,n} + 2A_{o,n}C_{o,n}] \\
&+ 2(-1)^{n+1} s^2 [A_{1,s}C_{1,s} + 2A_{o,s}C_{o,s}] \\
&+ (s-n)^2 [A_{1,n+s}C_{1,n+s} + 2A_{o,|n-s|}C_{o,|n-s|}] \\
&+ (s+n)^2 [2A_{o,n+s}C_{o,n+s} + W \cdot A_{1,|n-s|}C_{1,|n-s|}] \} \\
&- \sum_s (-1)^{s+2} \frac{8}{\pi} Q^* = 0
\end{aligned}
\tag{39}
$$

$n,s = 1,2,3....8$

where $W = 1.0$ if $n \neq s$

$W = 2.0$ if $n = s$

and

$$A_{pq} = \frac{\frac{3}{4}(1 - \nu_x \nu_y)}{(p^4 + 2\gamma\rho^2 p^2 q^2 + q^4 \rho^4)}$$

where

$$\phi_{pq} = A_{pq} C_{pq} \frac{\rho^4}{\theta J_y}$$

Equation (39) gives a set of eight simultaneous, non-linear equations for the eight coefficients b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16}, b_{17}, and b_{18}.

4.2 Computer Solution

Equation (39) is programmed on the IBM System 370 model 165 using Fortran IV level G1. The subroutine ZEROIN from the M.I.T. MATHLIB program library is used to solve the system of simultaneous, non-linear equations. This subprogram uses an iterative method of solution and convergence is achieved when the difference between two successive values of b_{nm} is less than 0.5×10^{-12}.

The foregoing analysis is for the completely general case with all the loadings as indicated in figure 1, however, the computer solutions were restricted to certain specific cases.

Basically, two programs were written to calculate the deflection coefficients. The first program produces curves of deflection at the center of the plate, effective width and bending moment in the y-direction at the center of the fixed supports versus inplane edge compressive load N_x^* with Q^* as a parameter, $N_y^* = 0$ and S^* fixed. Curves of this nature, for a range of orthotropic parameters γ and η, result in a set of design charts. The other program permits investigation of the behaviour for any set of loading conditions with $N_y^* = 0$. Deflection along the centerline $x = 0$ and total principal stresses along the centerlines $x = 0$ and $y = 0$ are plotted.

5. Results

5.1 Design Charts

The design program was used to generate a set of design charts with $N_y^* = S^* = 0$. Virtual plate aspect ratios of 1/1.5, 1/2, 1/4 and 1/6 are presented. For each aspect ratio a complete set of curves for orthotropic material parameters $\gamma = \eta = 1.0$ and $\gamma = 4.0$, $\eta = 0.5$ are given.

Description of design charts:

5.1.1 Charts of Deflection

The nondimensional deflection at the center of the plate, from equation (15) with $x = y = 0$, is plotted versus the inplane compressive edge load N_x^* with the lateral load Q^* as a parameter. Figures 2,3,8,9,14,15,20 and 21 give the deflection.

5.1.2 Charts of Effective Width

The nondimensional effective width given by equation (36b) is plotted versus the inplane compressive edge load N_x^* with the lateral load Q^* as a parameter. Figures 4,5,10, 11,16,17,22 and 23 give the effective width.

5.1.3 Charts of Bending Moment

The nondimensional bending moment M_y^* at the middle of the supports $y = \pm \frac{b}{2}$ and $x = 0$, equation (27), is plotted versus the inplane compressive edge load N_x^* with the lateral load Q^* as a parameter. Figures 6,7,12,13,18,19,24 and 25 give the bending moment.

5.2 Behaviour Charts

The behaviour program was used to investigate a few general sets of loading conditions with $N_y^* = 0$. Virtual plate aspect ratios of 1/1.5 and 1/1.25 are presented for orthotropic material properties $\gamma = \eta = 1.0$.

Description of behaviour charts:

5.2.1 Charts of Deflection

The nondimensional deflection, from equation (15), along the centerline of the plate (x = 0) is plotted versus y/b with either S* or N_x^* as a parameter and Q* fixed. Figures 26,27,36,37,46, and 47 give the deflection.

5.2.2 Charts of Maximum and Minimum Total Principal Stresses

The nondimensional total principal stresses, equation (37), along the centerlines on the bottom surface of the plate ($z= -\frac{h}{2}$) are plotted versus y/b (x = 0) and x/a (y = 0) with either S* or N_x^* as a parameter and Q* fixed. Figures 28-31, 38-41, and 48-51 give the stresses for x = 0 and Figures 32-35, 42-45, and 52-55 give the stresses for y = 0.

6. Discussion of Results

6.1 Orthotropic Properties

Reference [10] contains a complete discussion of the rigidity coefficients D_x, D_y and D_{xy} and compliance coefficients J_x, J_y and J_{xy} which are used to express the nondimensional parameters and orthotropic material coefficients. Possible approximate formulas for their calculatio are given in that reference. The two extreme cases of orthotropic plate coefficients are the isotropic case where $\eta = \gamma = 1.0$ and the grid (intersecting beams, no plate) $\eta = 0$ and $\gamma = \infty$. Most plate-stiffener combinations fall between $\eta = \gamma = 1.0$ and $\eta = 0.5$, $\gamma = 4.0$; hence the choice of parameters for the design charts.

Comparison of Figures 2 to 25 as to the nondimensional load parameters N_x^* and Q^* indicates as ρ decreases the assigned values of N_x^* decrease and Q^* increase. N_x^* and Q^* are indirect functions of the aspect ratio since they are nondimensionalized by the plate edges a and b respectively. The ranges of values used in the design curves were predicated on achieving a certain minimum value of effective breadth in the calculations.

6.2 Comparison with Existing Solutions

The first set of design curves to be produced were those for the case $\rho = \gamma = \eta = 1.0$, Figures 61,62 and 63, for N_y^* with Q^* as a parameter and $S^* = N_x^* = 0$. This choice of parameters permitted comparison with the solution obtaine

by Mansour [10]. The number of coefficients retained in this analysis (only one in the x-direction vice two by Mansour) does not favor the solution for the square plate case, however, the results of Figures 61, 62 and 63 agree to within 3% of the figures 1, 2 and 3 of Mansour [10]. The deflection coefficients obtained in reference [10] show that the one term assumption only neglects terms that are less than 5 1/2% of the primary deflection term b_{11}, which accounts for the reasonable agreement in this case.

For the loading conditions of the design curves, Figures 2-23, comparison was made with the solution of Levy [5]. Levy calculated deflection at the center of the plate and effective width for the isotropic plate of $\beta = 1.5$, $Q^* = S^* = 0$ and the same boundary conditions as this investigation. Comparison of Levy's results, Figure 3. and Table 11 of reference [5], with the curves for $Q^* = 0$ from Figures 2 and 3 is graphically displayed in Figures 64 and 65. Correlation of the two results is very good.

6.3 Design Charts

6.3.1 Charts of Deflection

The deflection at the center of the plate is always the maximum deflection since only the first deflection mode is encountered in these cases. With reference to the curve $Q^* = 0$ in the deflection charts, the plate remains undeflected until the critical compressive load N_x^* is reached. Increasing N_x^* further buckles the plate and plate deflection

increases rapidly. Hence, intersection of the curve $Q^* = 0$ and the N_x^* axis gives the lowest buckling load. The nonlinearity of the curves is evident. In the small deflection range, $\frac{w}{h} < \frac{1}{2}$, a doubling of the lateral load Q^* will double the deflection, however, in the large deflection range the lateral load effect becomes nonlinear with decreasing increments of plate deflection for increasing additions of load.

As ρ goes from 1/1.5 to 1/6 the magnitude of the maximum deflection of the plate is considerably reduced. The plate becomes more of a beam supported all around and hence "stiffer" than the slightly rectangular plate. The effect on the deflection of increasing Q^* as $\rho \rightarrow 1/6$ becomes less.

Comparison of buckling loads and the range of N_x^* values over the four aspect ratios shows that for $\rho = 1/1.5$ the maximum value of N_x^* considered is four times the buckling load while it is less than twice the buckling load for $\rho = 1/6$. The reason for this selection of N_x^* values is that the range where the solution is reliable decreases as $\rho \rightarrow 1/6$. Convergence to the correct solution becomes increasingly more difficult. In addition, the 8 term solution begins to become insufficient for $\rho = 1/6$ where the eight term, b_{18}, has increased to a value almost 6% of the primary term, b_{11}, for the highest set of N_x^* and Q^* considered.

6.3.2 Charts of Effective Width

By definition, the plate is fully effective in carrying the external compressive edge load until it buckles. With

reference to the effective width figures, $b_e/b = 1$ for $Q^* = 0$ until the buckling load is reached. Extensions of the curves for $Q^* \neq 0$ at values of N_x^* less than the buckling load are meaningless and should be ignored. After buckling, the effectiveness drops off as the plate deflects. Comparison of different aspect ratios shows that the effective width decreases more rapidly and takes on significantly lower values a $\rho \rightarrow 1/6$.

6.3.3 Charts of Bending Moment

The bending moment M_y^* at the middle of the fixed supports $y = \pm \frac{b}{2}$ is calculated because it is expected to be large. M_y^* is equal to $M_{y_o}^*$ at these points since the curvature along these supports in the x-direction is zero ($M_{x_o}^* = 0$), hence equation (28) is plotted. With reference to the bending moment figures, the bending moment is zero until the plate deflects. Increasing Q^* increases the bending moment in all cases with the nonlinearity associated with large deflections.

6.4 Behaviour Charts

Unfortunately, there are no existing solutions to which the results of shear loading may be compared for this particular set of boundary conditions. Levy [7,8] and Payer [11] considered the simply supported isotropic case. However, the results obtained are consistent with those anticipated in the magnitude and shape of the center line

deflection curve and values of principal stresses.

Results are given only for $\rho = 1/1.5$ and $1/1.25$. Investigation of the solution revealed that for virtual aspect ratios greater than about 1/1.75, the plate will have one symmetric buckle about the diagonal while for smaller values of ρ antisymmetric buckling shapes are experienced. The choice of one term in the x-direction limits the solution to some degree in the former case*, however, it cannot begin to approximate the true solution in the later case. Hence, the narrow range of investigation for ρ.

Additionally, with reference to the nondimensionalizing of the total stresses, sections 3.3 and 3.5, to present the results in a concise and meaningful form, the solution is restricted to the isotropic case where $\gamma = \eta = 1.0$. This is a result of the isotropic aspect ratio β appearing explicitly in the nondimensional equations. Nondimensionalizing the bending and membrane stresses to eliminate β will result in a set of incompatable nondimensional stresses which must be plotted and evaluated separately, thus increasing the number of charts required for each specific case.

Initially, the total principal stresses at both the top and bottom ($z = \pm \frac{h}{2}$ respectively) of the plate were investigated. Figures 56-60 show the results for $\rho = 0.667$, $\gamma = \eta = 1.0$, $S^* = 14$ (above critical load) and $N^* = Q^* = 0$. Using the Von Mises yield criteria for the biaxial case, $\sigma_z = 0$,

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*Note: The assumption of the one team solution in the x-direction will model a symmetric buckle about the plate centerline, not the diagonal.

$$(\sigma_0^*)^2 = (\sigma_1^*)^2 + (\sigma_2^*)^2 - \sigma_1^* \sigma_2^* \tag{40}$$

a combined stress may be calculated to evaluate the relative importance of the total stress field at the top or bottom of the plate. With reference to Figures 57-60, the table of $(\sigma_0^*)^2$ is as follows:

	y=x=0	$y=\pm\frac{b}{2}$, x=0	$x=\pm\frac{b}{2}$, y=0
Top	12.4	17.7	10.8
Bottom	12.2	19.5	10.8

The combined stresses are slightly greater at the top than at the bottom for the center of the plate, however, the magnitude and difference is considerably greater at the fixed supports with the bottom stresses being of higher value. By the nature of the upwards (+z) buckling mode, this is the expected result. Due to the higher combined stress at $y = \pm \frac{b}{2}$, the behaviour charts were plotted only for the principal stresses at the bottom of the plate $(z = -\frac{h}{2})$.

Figures 26-45 show the effect of increasing shear loads above the critical load for $\rho = 1/1.5$ and $1/1.25$. Inplane edge loads $N_x^* = N_y^* = 0$ and cases are presented for both $Q^* = 0$ and a finite value. Increasing S^* and/or Q^* increases the deflection and total principal stresses throughout the plate.

Figures 46-55 show the effects of completely general sets of loading conditions with increasing inplane edge compressive loads N_x^* above the critical load. Comparison of the charts for the sub-critical ($S^* = 6$) and super-critical

($S^* = 14$) shear loads clearly indicate the influence of shear on the shape of the deflection curve and values of principal stress. Increased shear increases deflection and principal stresses. In addition, the highest maximum principal stresses are observed to occur at the fixed support, $y = \pm \frac{b}{2}$.

6.5 Examples Demonstrating Use of the Charts

The following examples are given to demonstrate the use of the design and behaviour charts. The approximate formulas for calculation of the rigidity and compliance coefficients are discussed in detail in reference [10]. The formulas are listed in Table 2.

6.5.1 Design Example

Consider the following orthotropic characteristics and nondimensional loads for the stiffened plate:

$$\rho = 0.5,\ \gamma = 4.0,\ \eta = 0.5,\ N_x^* = 4.0,\ Q^* = S^* = 0$$

From Figure 9

The critical load $\bar{N}_{x_c} = 1.8\frac{\pi^2 D_x}{a^2}$

The center of the plate deflection

$$w = 1.64h$$

where h is the average stiffener's depth plus the plate thickness.

From Figure 11

The effective width $b_e = 0.505b$

Edge membrane stress (equations 24,34b and 35b) is

$$\sigma^*_{x_e} \simeq -\left(\frac{b}{b_e}\right) \frac{N^*_x}{12(1-\nu^2)} = -\frac{1}{0.505} \cdot \frac{4.0}{10.9}$$

$$\sigma_{x_e} \simeq 0.73 \frac{\pi^2 h}{a^2 J_y}$$

From Figure 13

The bending moment at the middle of the fixed support

$$M^*_y = -102$$

The maximum bending stress at this point (equation 29) is

$$\sigma_{b_y} = \pm \frac{6M^*_y}{h^2} \cdot \frac{hD_y}{b^2} = \mp 612 \frac{D_y}{hb^2}$$

6.5.2 Behaviour Example

Consider the following orthotropic characteristics and nondimensional loads for an isotropic plate:

$\rho = 0.667$, $\gamma = \eta = 1.0$, $N^*_x = 3.0$, $Q^* = 8.0$, $S^* = 6.0$

From Figure 46

The maximum deflection at the center of the plate $w = 1.74\ h$

From Figures 48 and 50

The maximum principal stresses occur at the fixed supports $y = \pm \frac{b}{2}$, $x = 0$ and are

$$\sigma_1 = 6.50 \frac{\pi^2 h}{b^2 J_x}$$

$$\sigma_2 = -1.54 \frac{\pi^2 h}{b^2 J_x}$$

and the combined stress

$$\sigma_0^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = 54.6\ \left[\frac{\pi^2 h}{b^2 J_x}\right]^2$$

7. Conclusions and Recommendations

The problem of a large aspect ratio orthotropic plate with the short edges fixed and long edges simply supported has been modeled for the case of inplane edge compressive and lateral loads and subcritical edge shear loads. The design charts provide valuable information of use in establishing design criteria for the plate-stiffener combination.

From the behaviour Figures 46 to 55, it is evident that the highest principal stresses occur at the fixed supports $y = \pm \frac{b}{2}$. The design Figures 2 to 25 do not present information to permit the calculation of the membrane stresses at the fixed supports. Design charts of σ_{y_e}, edge membrane stress in the y-direction (denominator of equation (36a)), should be produced.

As noted in the results, the one term assumption in the x-direction will not model the deflection of a plate with edge shear as the predominate loading. As the major point of this investigation was to model a large aspect ratio plate, the solution chosen was an inevitable consequence when the large range of ρ is considered. It is recommended that to model a large aspect ratio plate subjected to shear loading, close attention must be paid to the important deflection terms that should be included in the solution. Levy [8] confined himself to a $\beta = 1.5$ and used 14 selected terms in the deflection equation.

Additionally, the eight term solution is just sufficient to model the case for the orthotropic plate aspect ratio of 1/6.

8. References

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TABLE 1 - COEFFICIENTS C_{pq}

$$C_{pq} = Y_q + (-1)^p X_q$$

$$p = 0,1$$

$$q = 0,1,2\ldots16$$

where X_q and Y_q are given by

$$X_o = b_{11}^2 + 4b_{12}^2 + 9b_{13}^2 + 16b_{14}^2 + 25b_{15}^2 + 36b_{16}^2 + 49b_{17}^2 + 64b_{18}^2$$

$$Y_o = -X_o$$

$$X_1 = 4b_{11}b_{12} + 12b_{12}b_{13} + 24b_{13}b_{14} + 40b_{14}b_{15} + 60b_{15}b_{16}$$
$$+ 84b_{16}b_{17} + 112b_{17}b_{18}$$

$$Y_1 = 2b_{11}(-b_{11} - \tfrac{3}{2}b_{12} - b_{13} + b_{14} - b_{15} + b_{16} - b_{17} + b_{18})$$
$$-13b_{12}b_{13} - 25b_{13}b_{14} - 41b_{14}b_{15} - 61b_{15}b_{16} - 85b_{16}b_{17} - 113b_{17}b_{18}$$

$$X_2 = -b_{11}^2 + 6b_{11}b_{13} + 16b_{12}b_{14} + 30b_{13}b_{15} + 48b_{14}b_{16}$$
$$+ 70b_{15}b_{17} + 96b_{16}b_{18}$$

$$Y_2 = 8b_{12}(-b_{11} + b_{12} - b_{13} - \tfrac{3}{2}b_{14} - b_{15} + b_{16} - b_{17} + b_{18})$$
$$-b_{11}^2 - 10b_{11}b_{13} - 34b_{13}b_{15} - 52b_{14}b_{16} - 74b_{15}b_{17}$$
$$- 100b_{16}b_{18}$$

$$X_3 = -4b_{11}b_{12} + 8b_{11}b_{14} + 20b_{12}b_{15} + 36b_{13}b_{16} + 56b_{14}b_{17}$$
$$+ 80b_{15}b_{18}$$

$$Y_3 = 18b_{13}(-b_{11} + b_{12} - b_{13} + b_{14} - b_{15} - \tfrac{3}{2}b_{16} - b_{17} + b_{18})$$
$$-5b_{11}b_{12} - 17b_{11}b_{14} - 29b_{12}b_{15} - 65b_{14}b_{17} - 89b_{15}b_{18}$$

$$X_4 = -6b_{11}b_{13} + 10b_{11}b_{15} - 4b_{12}^2 + 24b_{12}b_{16} + 42b_{13}b_{17}$$
$$+ 64b_{14}b_{18}$$
$$Y_4 = 32b_{14}(-b_{11} + b_{12} - b_{13} + b_{14} - b_{15} + b_{16} - b_{17} - \tfrac{3}{2}b_{18})$$
$$-10b_{11}b_{13} - 26b_{11}b_{15} - 4b_{12}^2 - 40b_{12}b_{16} - 58b_{13}b_{17}$$

$$X_5 = -8b_{11}b_{14} + 12b_{11}b_{16} - 12b_{12}b_{13} + 28b_{12}b_{17} + 48b_{13}b_{18}$$
$$Y_5 = 50b_{15}(-b_{11} + b_{12} - b_{13} + b_{14} - b_{15} + b_{16} - b_{17} + b_{18})$$
$$-17b_{11}b_{14} - 37b_{11}b_{16} - 13b_{12}b_{13} - 53b_{12}b_{17} - 73b_{13}b_{18}$$

$$X_6 = -10b_{11}b_{15} + 14b_{11}b_{17} - 16b_{12}b_{14} + 32b_{12}b_{18} - 9b_{13}^2$$
$$Y_6 = 72b_{16}(-b_{11} + b_{12} - b_{13} + b_{14} - b_{15} + b_{16} - b_{17} + b_{18})$$
$$-26b_{11}b_{15} - 50b_{11}b_{17} - 20b_{12}b_{14} - 68b_{12}b_{18} - 9b_{13}^2$$

$$X_7 = -12b_{11}b_{16} + 16b_{11}b_{18} - 20b_{12}b_{15} - 24b_{13}b_{14}$$
$$Y_7 = 98b_{17}(-b_{11} + b_{12} - b_{13} + b_{14} - b_{15} + b_{16} - b_{17} + b_{18})$$
$$-37b_{11}b_{16} - 65b_{11}b_{18} - 29b_{12}b_{15} - 25b_{13}b_{14}$$

$$X_8 = -14b_{11}b_{17} - 24b_{12}b_{16} - 30b_{13}b_{15} - 16b_{14}^2$$

$$Y_8 = 128b_{18}(-b_{11} + b_{12} - b_{13} + b_{14} - b_{15} + b_{16} - b_{17} + b_{18})$$
$$-50b_{11}b_{17} - 40b_{12}b_{16} - 34b_{13}b_{15} - 16b_{14}^2$$

$$X_9 = -16b_{11}b_{18} - 28b_{12}b_{17} - 36b_{13}b_{16} - 40b_{14}b_{15}$$
$$Y_9 = -65b_{11}b_{18} - 53b_{12}b_{17} - 45b_{13}b_{16} - 41b_{14}b_{15}$$

$$X_{10} = -32b_{12}b_{18} - 42b_{13}b_{17} - 48b_{14}b_{16} - 25b_{15}^2$$
$$Y_{10} = -68b_{12}b_{18} - 58b_{13}b_{17} - 52b_{14}b_{16} - 25b_{15}^2$$

$$X_{11} = -48b_{13}b_{18} - 56b_{14}b_{17} - 60b_{15}b_{16}$$
$$Y_{11} = -73b_{13}b_{18} - 65b_{14}b_{17} - 61b_{15}b_{16}$$

$$X_{12} = -64b_{14}b_{18} - 70b_{15}b_{17} - 36b_{16}^2$$
$$Y_{12} = -80b_{14}b_{18} - 74b_{15}b_{17} - 36b_{16}^2$$

$$X_{13} = -80b_{15}b_{18} - 84b_{16}b_{17}$$
$$Y_{13} = -89b_{15}b_{18} - 85b_{16}b_{17}$$

$$X_{14} = -96b_{16}b_{18} - 49b_{17}^2$$
$$Y_{14} = -100b_{16}b_{18} - 49b_{17}^2$$

$$X_{15} = -112b_{17}b_{18}$$
$$Y_{15} = -113b_{17}b_{18}$$

$X_{16} = -64b_{18}^{2}$

$Y_{16} = X_{16}$

TABLE 2 - RIGIDITY AND COMPLIANCE COEFFICIENTS

$$D_x \simeq \frac{EI_x}{S_x(1-\nu^2)}$$

$$D_y \simeq \frac{EI_y}{S_y(1-\nu^2)}$$

$$\rho \simeq \frac{a}{b}\sqrt[4]{\frac{I_y}{I_x}\frac{S_x}{S_y}}$$

$$\eta = \frac{D_{xy}}{\sqrt{D_x D_y}} \simeq \sqrt{\frac{I_{px} I_{py}}{I_x I_y}}$$

$$J_x \simeq \frac{1}{Eh_y}$$

$$J_y \simeq \frac{1}{Eh_x}$$

$$J_{xy} \simeq \frac{1}{E}\left[\frac{1+\nu}{h_p} - \frac{\nu}{\bar{h}}\right]$$

$$\gamma = \frac{J_{xy}}{\sqrt{J_x J_y}} \simeq (1 + \nu)\frac{\sqrt{h_x h_y}}{h_p} - \nu\frac{\sqrt{h_x h_y}}{\bar{h}}$$

$$\bar{h} = \frac{2h_x h_y}{h_x + h_y}$$

TABLE 3 - INTEGRALS

$$\int_0^{b/2}\int_0^{a/2} \cos\frac{m\pi}{a}x \cos\frac{r\pi}{a}x \, dxdy = \frac{ab}{8} \qquad m = r$$
$$= 0 \qquad m \neq r$$

$$\int_0^{b/2}\int_0^{a/2} \cos\frac{m\pi}{a}x \cos\frac{r\pi}{a}x \cos\frac{2s\pi}{b}y \, dxdy = 0$$

$$\int_0^{b/2}\int_0^{a/2} \cos\frac{m\pi}{a}x \cos\frac{r\pi}{a}x \cos\frac{2n\pi}{b}y \, dxdy = 0$$

$$\int_0^{b/2}\int_0^{a/2} \cos\frac{m\pi}{a}x \cos\frac{r\pi}{a}x \cos\frac{2n\pi}{b}y \cos\frac{2s\pi}{b}y \, dxdy = \frac{ab}{16}, \; m=r, \; n=s$$
$$= 0 \quad m\neq r \text{ or } n\neq s$$

$$\int_0^{b/2}\int_0^{a/2} \cos\frac{m\pi}{a}x \cos\frac{2p\pi}{a}x \cos\frac{r\pi}{a}x \cos\frac{2q\pi}{b}y \, dxdy = 0$$

$$\int_0^{b/2}\int_0^{a/2} \cos\frac{r\pi}{a}x \cos\frac{2s\pi}{b}y \, dxdy = 0$$

$$\int_0^{b/2}\int_0^{a/2} \cos\frac{r\pi}{a}x \, dxdy = \frac{ab}{2r\pi}(-1)^{\frac{r-1}{2}} \qquad r \text{ odd}$$
$$= 0 \qquad \text{otherwise}$$

$$\int_0^{b/2}\int_0^{a/2} \cos\frac{m\pi}{a}x \cos\frac{2p\pi}{a}x \cos\frac{r\pi}{a}x \cos\frac{2q\pi}{b}y \cos\frac{2s\pi}{b}y \, dxdy$$
$$= \frac{ab}{32}, \; m+r = 2p, \; q = s$$
$$= \frac{ab}{32}, \; |m-r| = 2p, \; q=s, \; m > r$$
$$= \frac{ab}{32}, \; |r-m| = 2p, \; q=s, \; r > m$$
$$= 0 \quad \text{otherwise}$$

$$\int_0^{b/2}\int_0^{a/2} \cos\frac{m\pi}{a}x \, \cos\frac{2p\pi}{a}x \, \cos\frac{r\pi}{a}x \, \cos\frac{2n\pi}{b}y \, \cos\frac{2q\pi}{b}y \, dxdy$$

$$= \frac{ab}{32}, \quad m + r = 2p \, , \; q = n$$

$$= \frac{ab}{32}, \quad |m - r| = 2p, \; q = n, \; m > r$$

$$= \frac{ab}{32}, \quad |r - m| = 2p, \; q = n, \; r > m$$

$$= 0 \quad \text{otherwise}$$

$$\int_0^{b/2}\int_0^{a/2} \sin\frac{m\pi}{a}x \, \sin\frac{2p\pi}{a}x \, \cos\frac{r\pi}{a}x \, \sin\frac{2n\pi}{b}y \, \sin\frac{2q\pi}{b}y \, dxdy$$

$$= \frac{ab}{32}, \quad m + r = 2p \, , \; n = q$$

$$= \frac{ab}{32}, \quad |m - r| = 2p, \; n=q, \; m > r$$

$$= -\frac{ab}{32}, \quad |r - m| = 2p, \; n=q, \; r > m$$

$$= 0 \quad \text{otherwise}$$

$$\int_0^{b/2}\int_0^{a/2} \cos\frac{m\pi}{a}x \, \cos\frac{2p\pi}{a}x \, \cos\frac{r\pi}{a}x \, \cos\frac{2n\pi}{b}y \, \cos\frac{2q\pi}{b}y \, \cos\frac{2s\pi}{b}y \, dxdy$$

$$= \frac{ab}{64}, \quad m + r = 2p, \; n + s = q$$

$$= \frac{ab}{64}, \quad m + r = 2p, \; |n - s| = q, \; n > s$$

$$= \frac{ab}{64}, \quad m + r = 2p, \; |s - n| = q, \; s > n$$

$$= \frac{ab}{64}, \quad |m - r| = 2p, \; n + s = q, \; m > r$$

$$= \frac{ab}{64}, \quad |m - r| = 2p, \; |n - s| = q, \; m>r, \; n>s$$

$$= \frac{ab}{64}, \quad |m - r| = 2p, \; |s - n| = q, \; m>r, \; s>n$$

$$= \frac{ab}{64}, \quad |r - m| = 2p, \; n + s = q, \; r > m$$

$$= \frac{ab}{64}, \quad |r - m| = 2p, |n - s| = q, \ r>m, \ n>s$$

$$= \frac{ab}{64}, \quad |r - m| = 2p, |s - n| = q, \ r>m, \ s>n$$

$$= 0 \quad \text{otherwise}$$

$$\int_0^{b/2}\int_0^{a/2} \sin\frac{m\pi}{a}x \ \sin\frac{2p\pi}{a}x \ \cos\frac{r\pi}{a}x \ \sin\frac{2n\pi}{b}y \ \sin\frac{2q\pi}{b}y \ \cos\frac{2s\pi}{b}y \ dxdy$$

$$= \frac{ab}{64}, \quad m + r = 2p, \ n + s = q$$

$$= \frac{ab}{64}, \quad m + r = 2p, |n - s| = q, \ n > s$$

$$= -\frac{ab}{64}, \quad m + r = 2p, |s - n| = q, \ s > n$$

$$= \frac{ab}{64}, \quad |m - r| = 2p, \ n + s = q, \ m > r$$

$$= \frac{ab}{64}, \quad |m - r| = 2p, |n - s| = q, \ m>r, \ n>s$$

$$= -\frac{ab}{64}, \quad |m - r| = 2p, |s - n| = q, \ m>r, \ s>n$$

$$= -\frac{ab}{64}, \quad |r - m| = 2p, \ n + s = q, \ r > m$$

$$= -\frac{ab}{64}, \quad |r - m| = 2p, |n - s| = q, \ r>m, \ n>s$$

$$= \frac{ab}{64}, \quad |r - m| = 2p, |s - n| = q, \ r>m, \ s>n$$

$$= 0 \quad \text{otherwise}$$

$$\int_0^{b/2}\int_0^{a/2} \sin\frac{m\pi}{a}x \ \cos\frac{r\pi}{a}x \ \sin\frac{2n\pi}{b}y \ dxdy$$

$$= \frac{ab}{n\pi^2}\left(\frac{1}{m+r}\right), \qquad \frac{m+r}{2} \text{ odd, } n \text{ odd}$$

$$= \frac{ab}{n\pi^2}\left(\frac{1}{m-r}\right), \qquad \frac{m-r}{2} \text{ odd, } n \text{ odd}$$

$$= 0 \qquad \text{otherwise}$$

$$\int_0^{b/2}\int_0^{a/2} \sin\frac{m\pi}{a}x \, \cos\frac{r\pi}{a}x \, \sin\frac{2n\pi}{b}y \, \cos\frac{2s\pi}{b}y \; dxdy$$

$$= \frac{ab}{2\pi^2}\left(\frac{1}{m+r}\right)\left(\frac{1}{n+s} + \frac{1}{n-s}\right), \quad \frac{m+r}{2} \text{ odd, } n+s \text{ odd}$$

$$= \frac{ab}{2\pi^2}\left(\frac{1}{m-r}\right)\left(\frac{1}{n+s} + \frac{1}{n-s}\right), \quad \frac{m-r}{2} \text{ odd, } n+s \text{ odd}$$

$$= 0 \quad \text{otherwise}$$

Table 4 - Computer Symbols

Fortran Symbol	Variable
B(N)	$b_{1,n}$
RHO	ρ
GAMMA	γ
ETA	η
QSTAR	Q^*
NXSTAR	N_x^*
NYSTAR	N_y^*
SSTAR	S^*
W/H	w/h
AE/A	a_e/a
BE/B	b_e/b
MYSTAR	M_y^*
SIGMA STAR	σ_1^* or σ_2^*
CPQ	C_{pq}
APQ	A_{pq}
RANG	Max. value of N^* or S^*
RANGQ	Max. value of Q^*

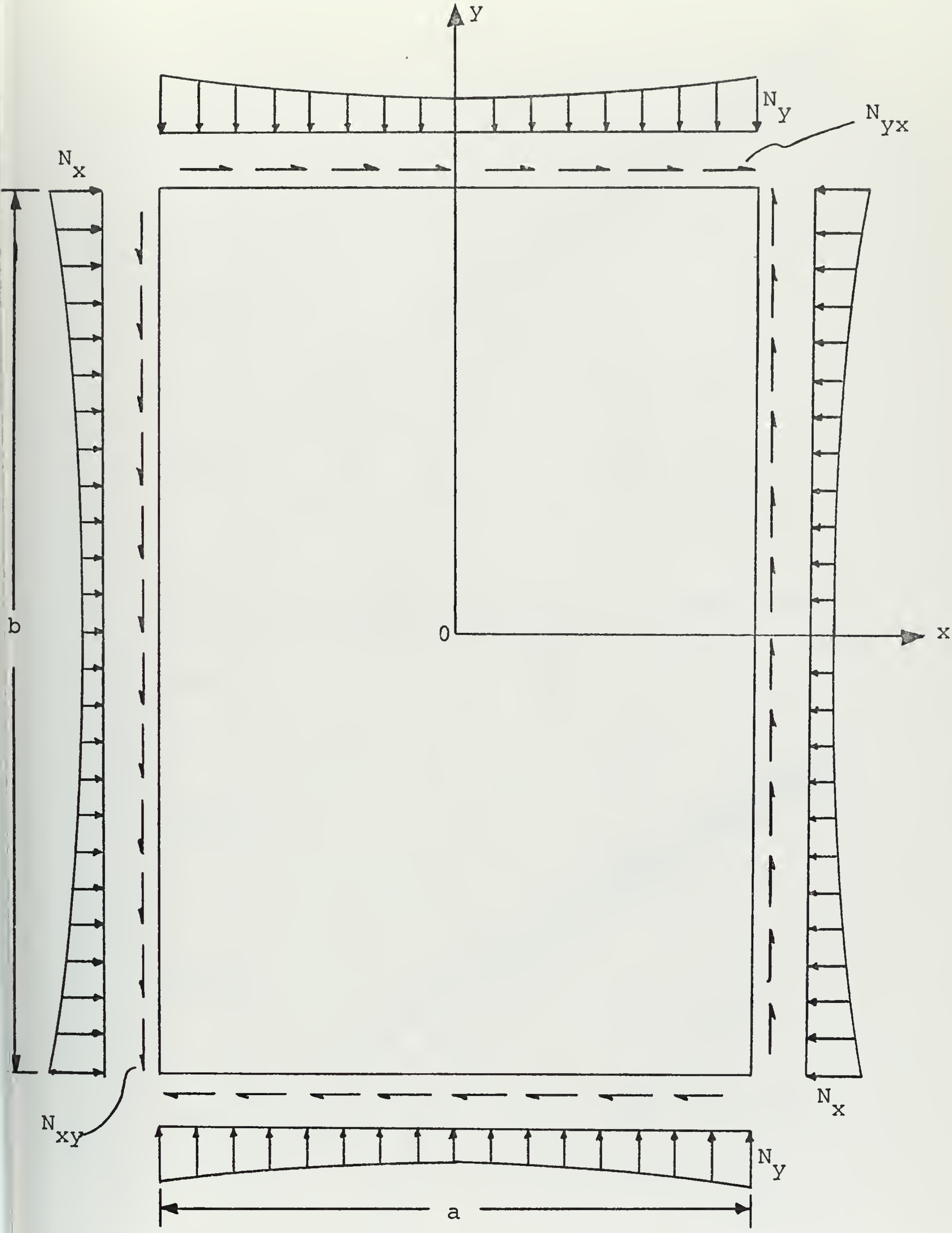
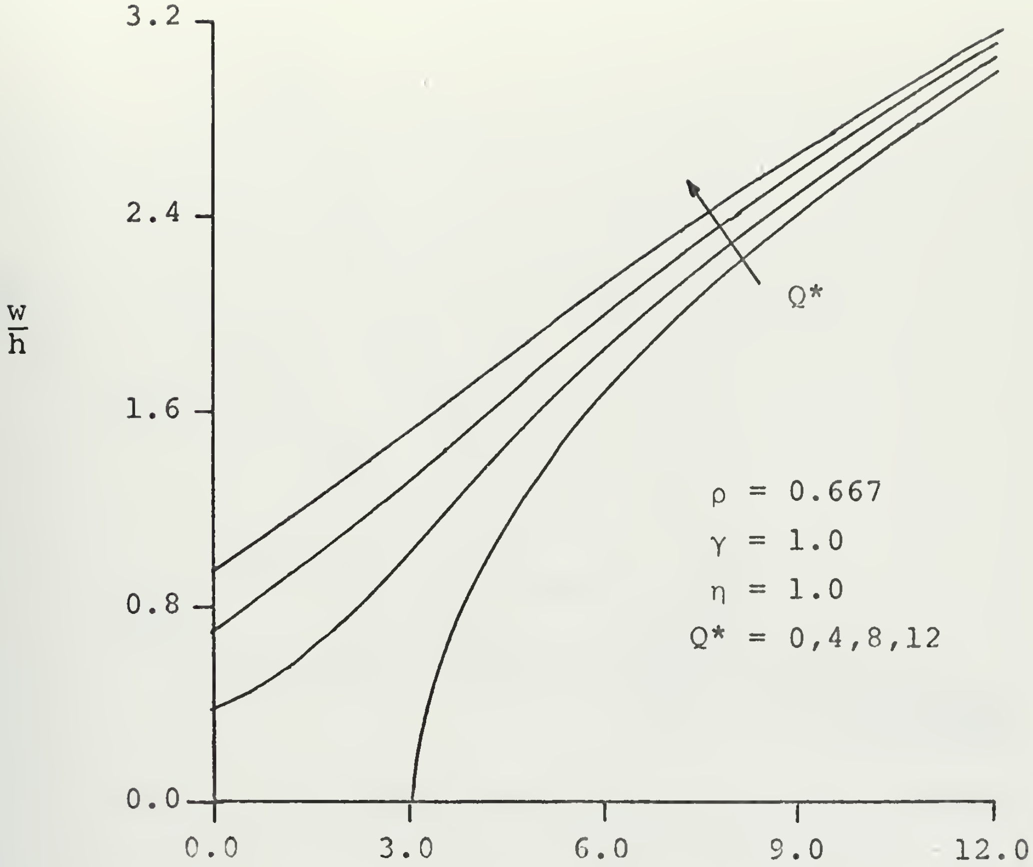
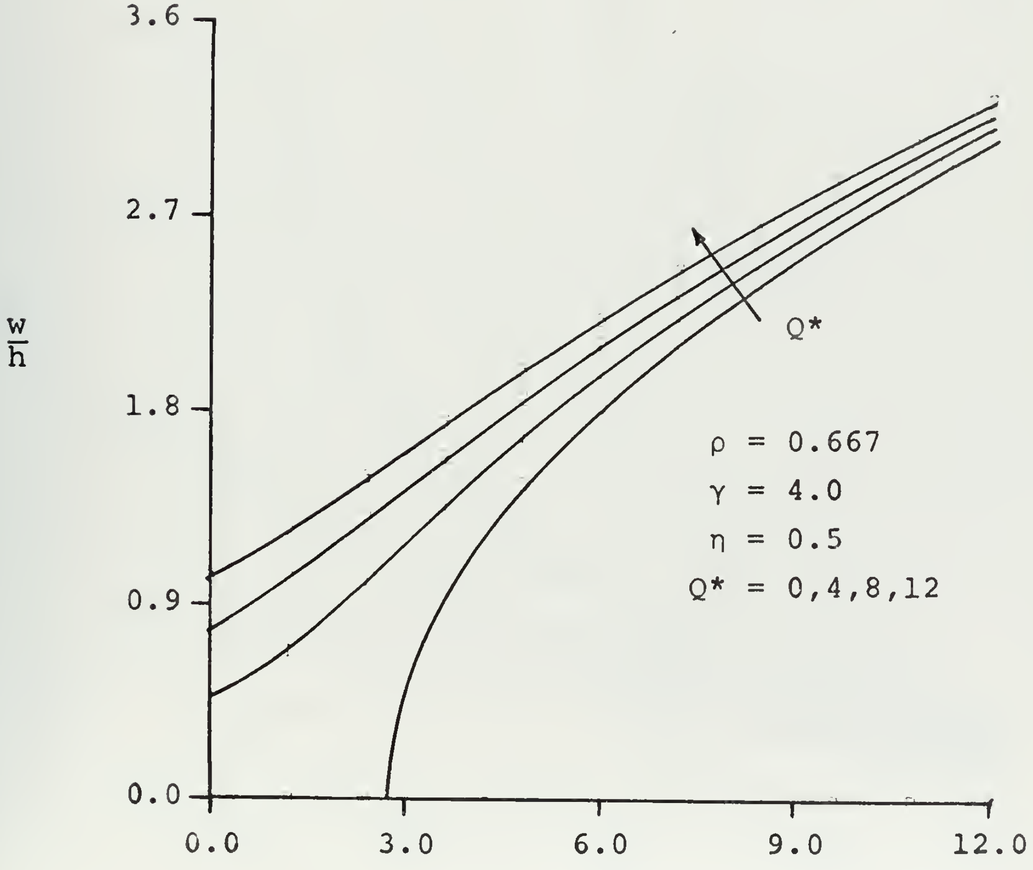
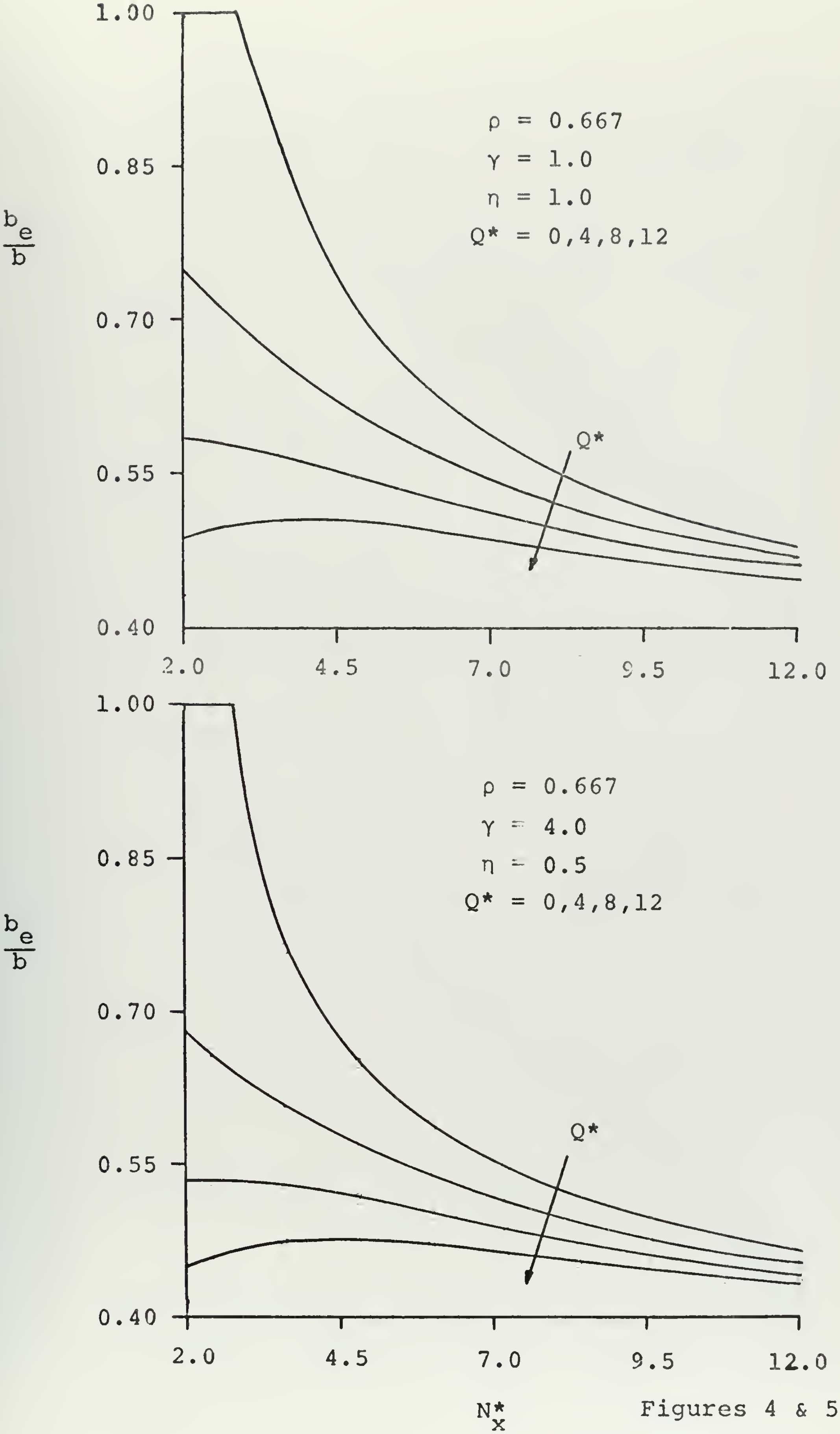


Figure 1

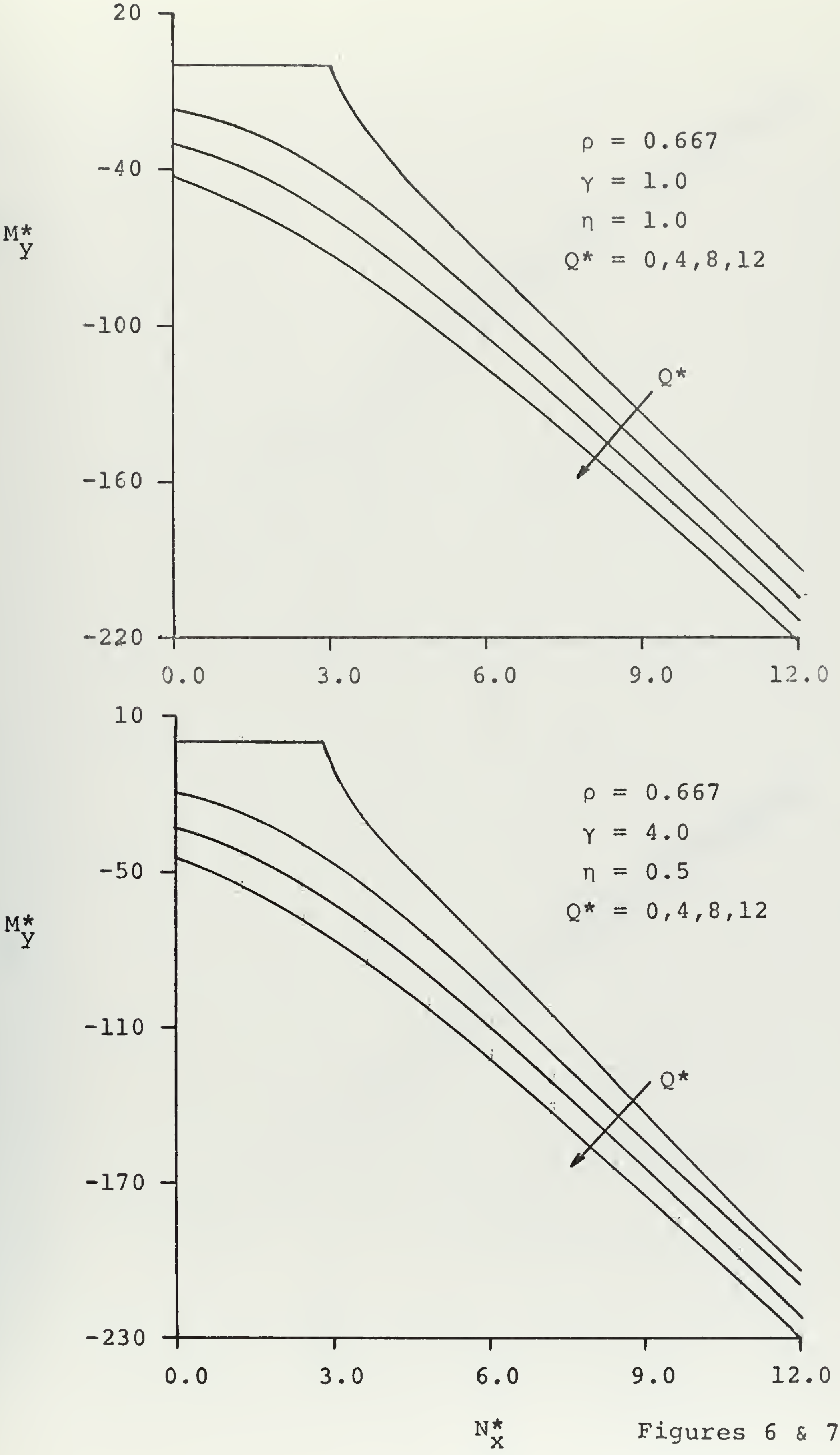

3.2
2.4
1.6
0.8
0.0
$\frac{w}{h}$
0.0
3.0
6.0
9.0
12.0
Q*
ρ = 0.667
γ = 1.0
η = 1.0
Q* = 0,4,8,12


3.6
2.7
1.8
0.9
0.0
$\frac{w}{h}$
0.0
3.0
6.0
9.0
12.0
N_x^*
Q*
ρ = 0.667
γ = 4.0
η = 0.5
Q* = 0,4,8,12

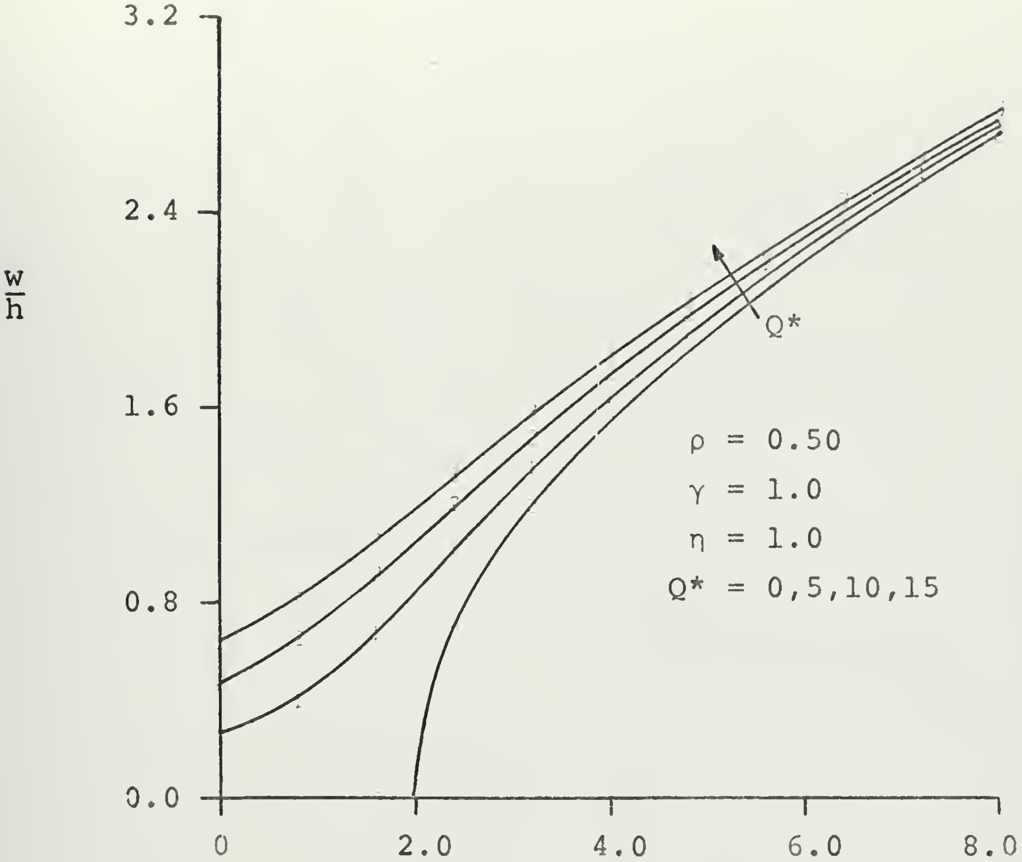
Figures 2 & 3

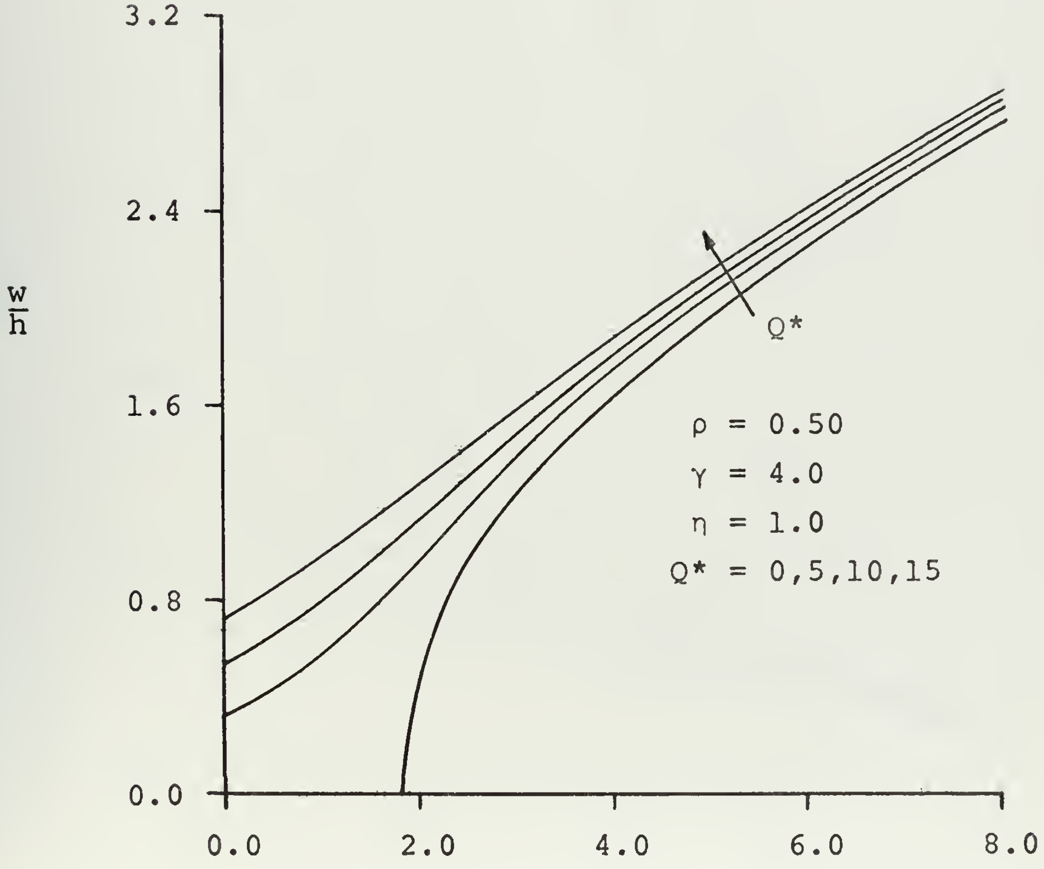

1.00
0.85
0.70
0.55
0.40
$\frac{b_e}{b}$
ρ = 0.667
γ = 1.0
η = 1.0
Q* = 0,4,8,12
Q*
2.0
4.5
7.0
9.5
12.0
1.00
0.85
0.70
0.55
0.40
$\frac{b_e}{b}$
ρ = 0.667
γ = 4.0
η = 0.5
Q* = 0,4,8,12
Q*
2.0
4.5
7.0
9.5
12.0
N_x^*

Figures 4 & 5

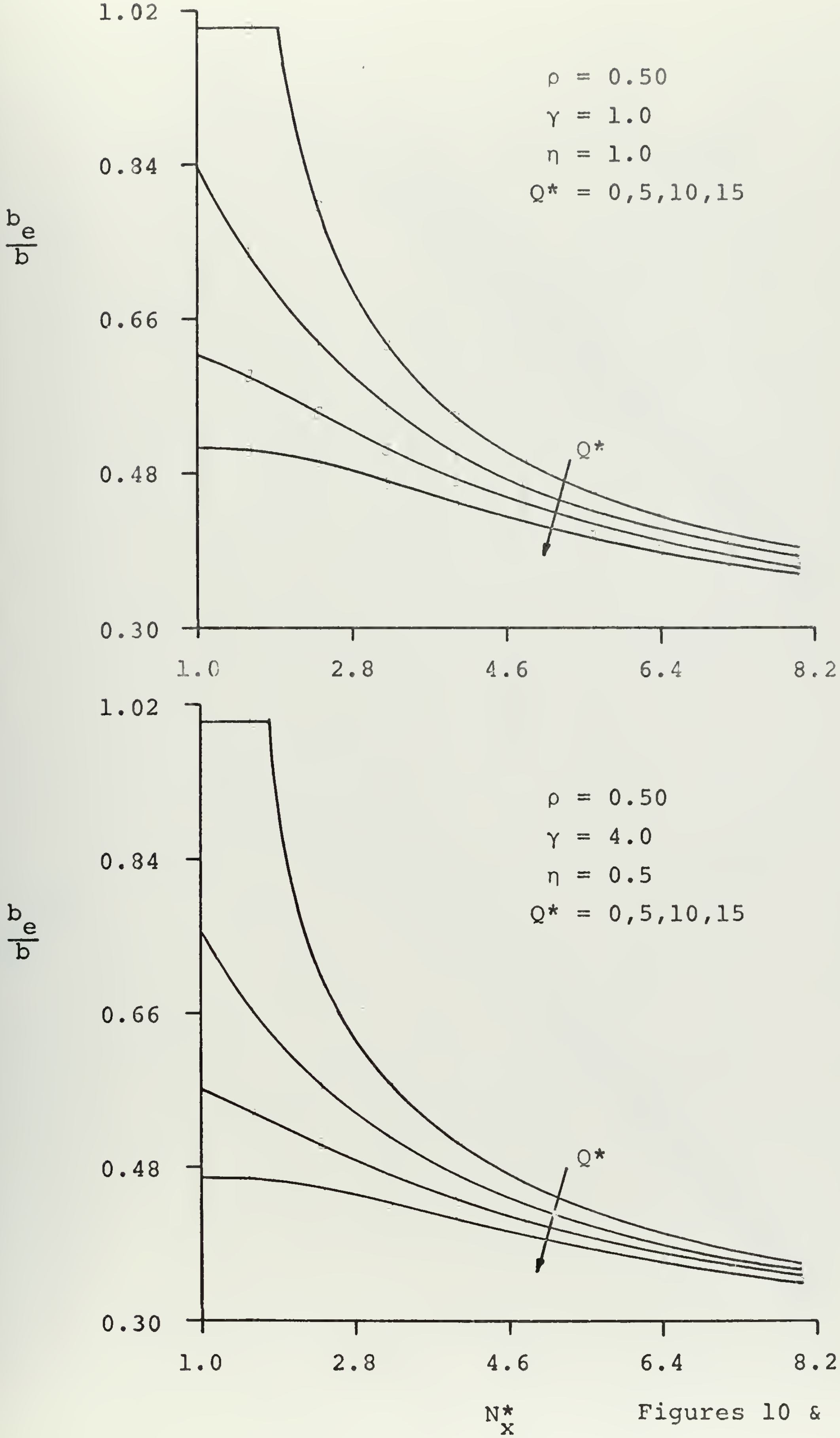

20
-40
-100
-160
-220
M_y^*
0.0
3.0
6.0
9.0
12.0
ρ = 0.667
γ = 1.0
η = 1.0
Q* = 0,4,8,12
Q*
10
-50
-110
-170
-230
M_y^*
0.0
3.0
6.0
9.0
12.0
ρ = 0.667
γ = 4.0
η = 0.5
Q* = 0,4,8,12
Q*
N_x^*

Figures 6 & 7

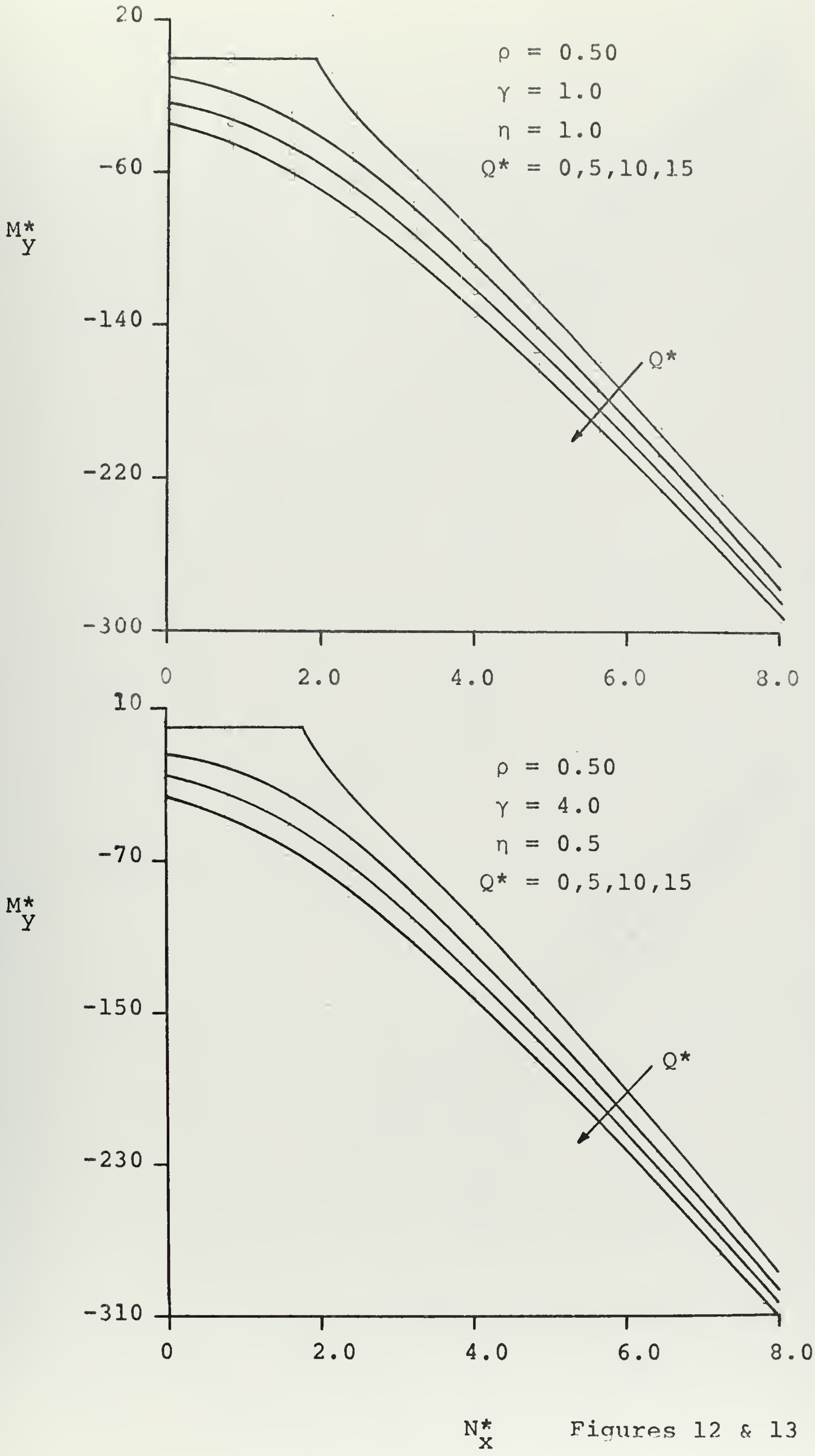

3.2
2.4
1.6
0.8
0.0
$\frac{w}{h}$
0
2.0
4.0
6.0
8.0
Q*
ρ = 0.50
γ = 1.0
η = 1.0
Q* = 0,5,10,15


3.2
2.4
1.6
0.8
0.0
$\frac{w}{h}$
0.0
2.0
4.0
6.0
8.0
Q*
ρ = 0.50
γ = 4.0
η = 1.0
Q* = 0,5,10,15
N_x^*

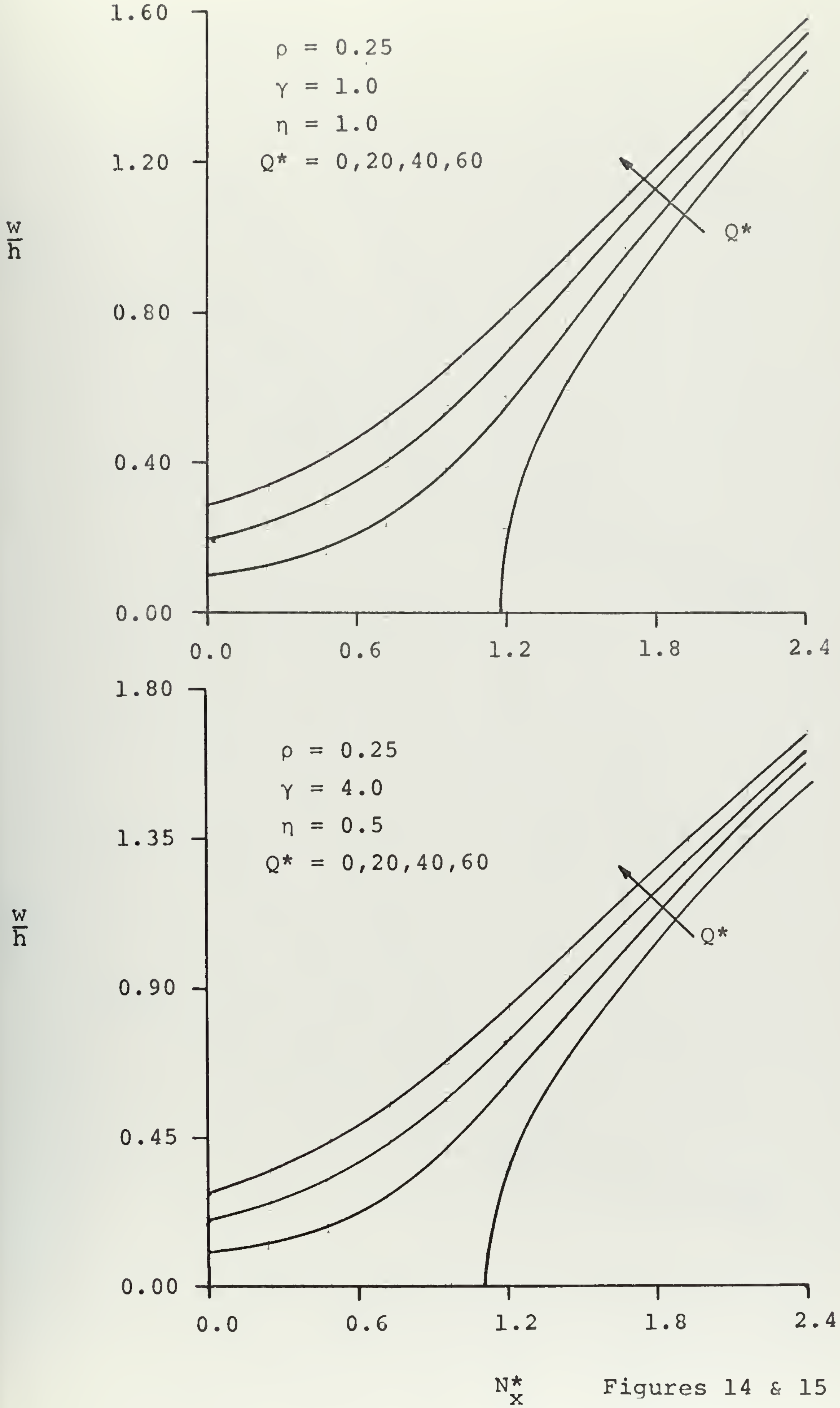
Figures 8 & 9



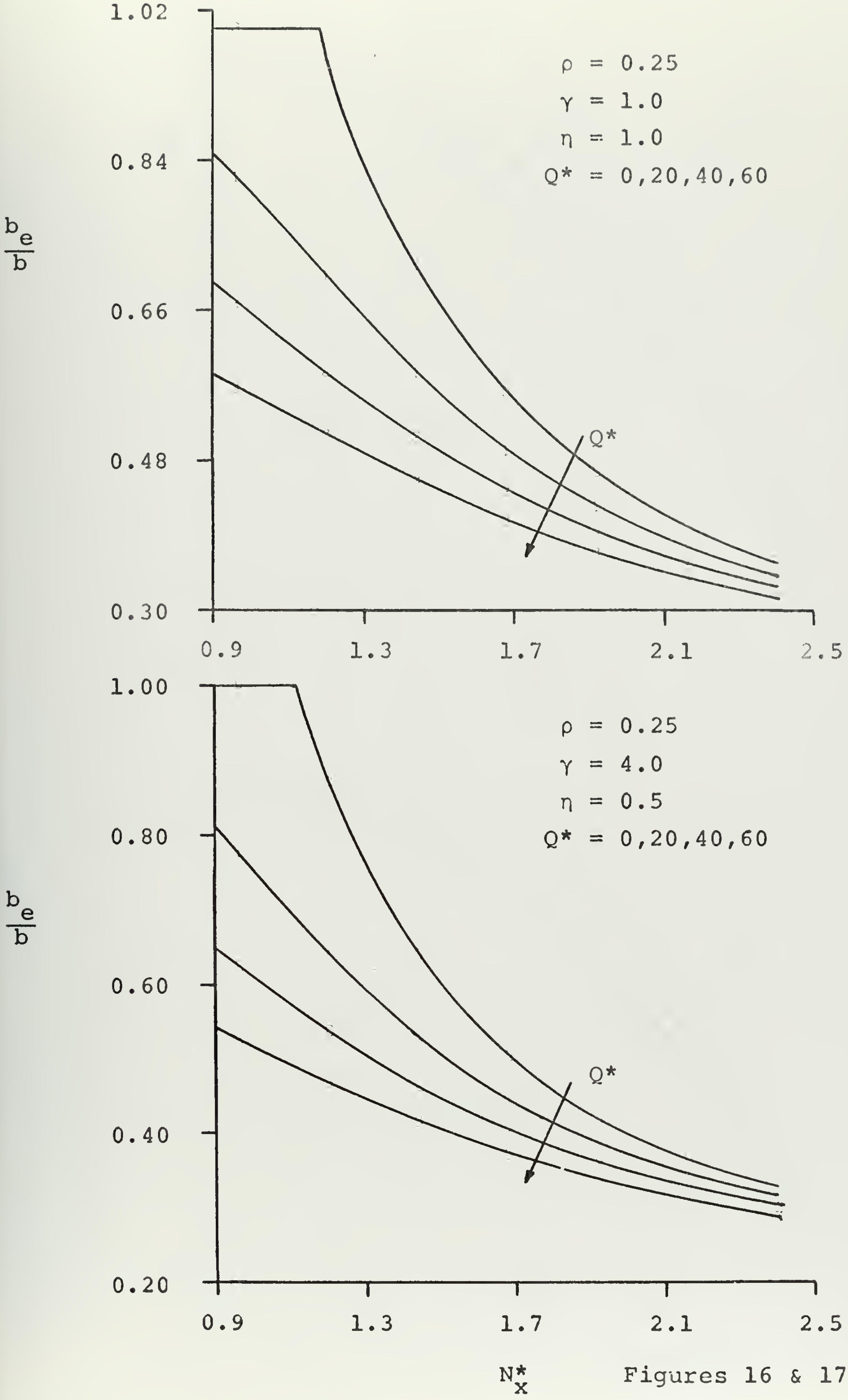
Figures 10 & 11



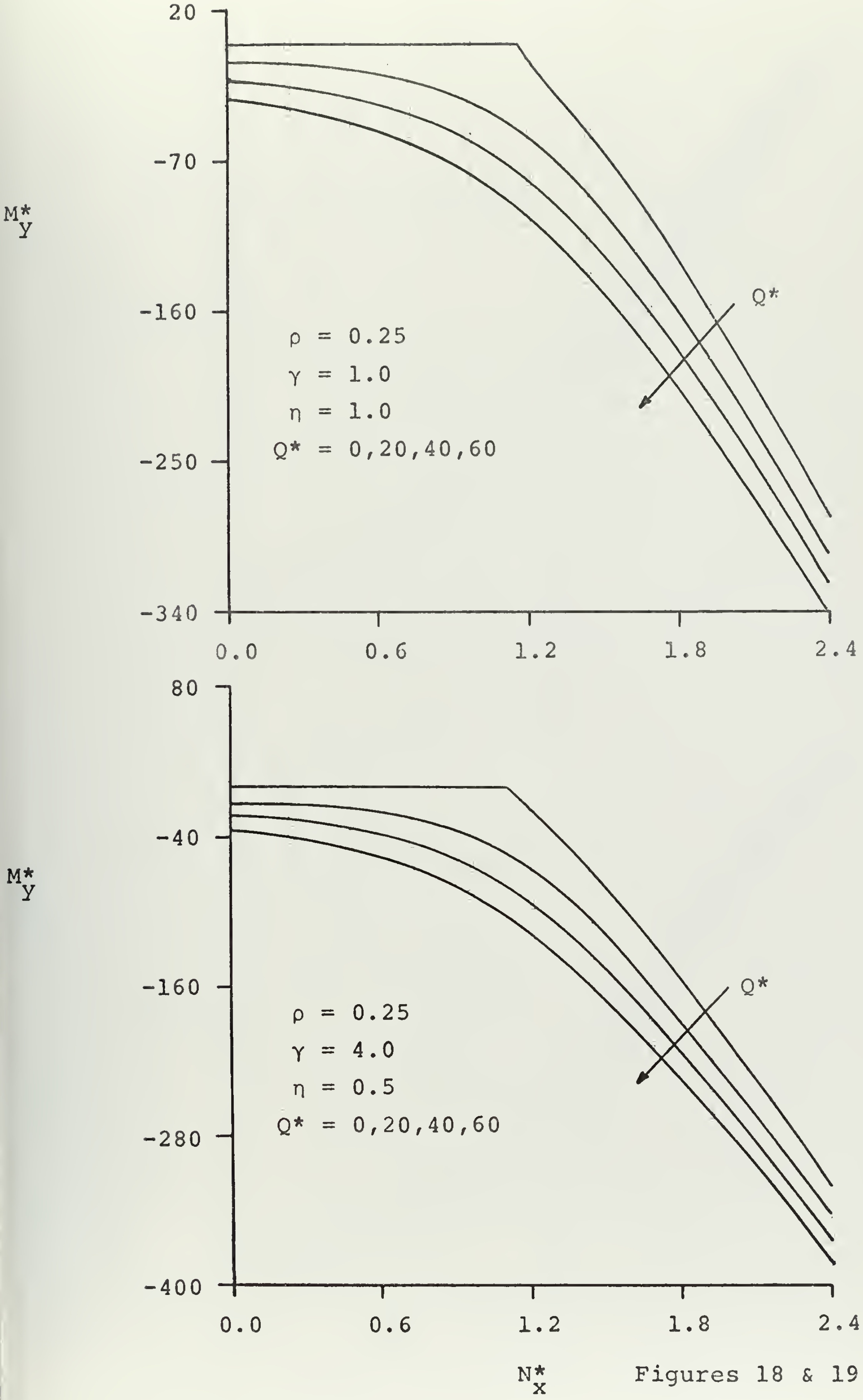
Figures 12 & 13



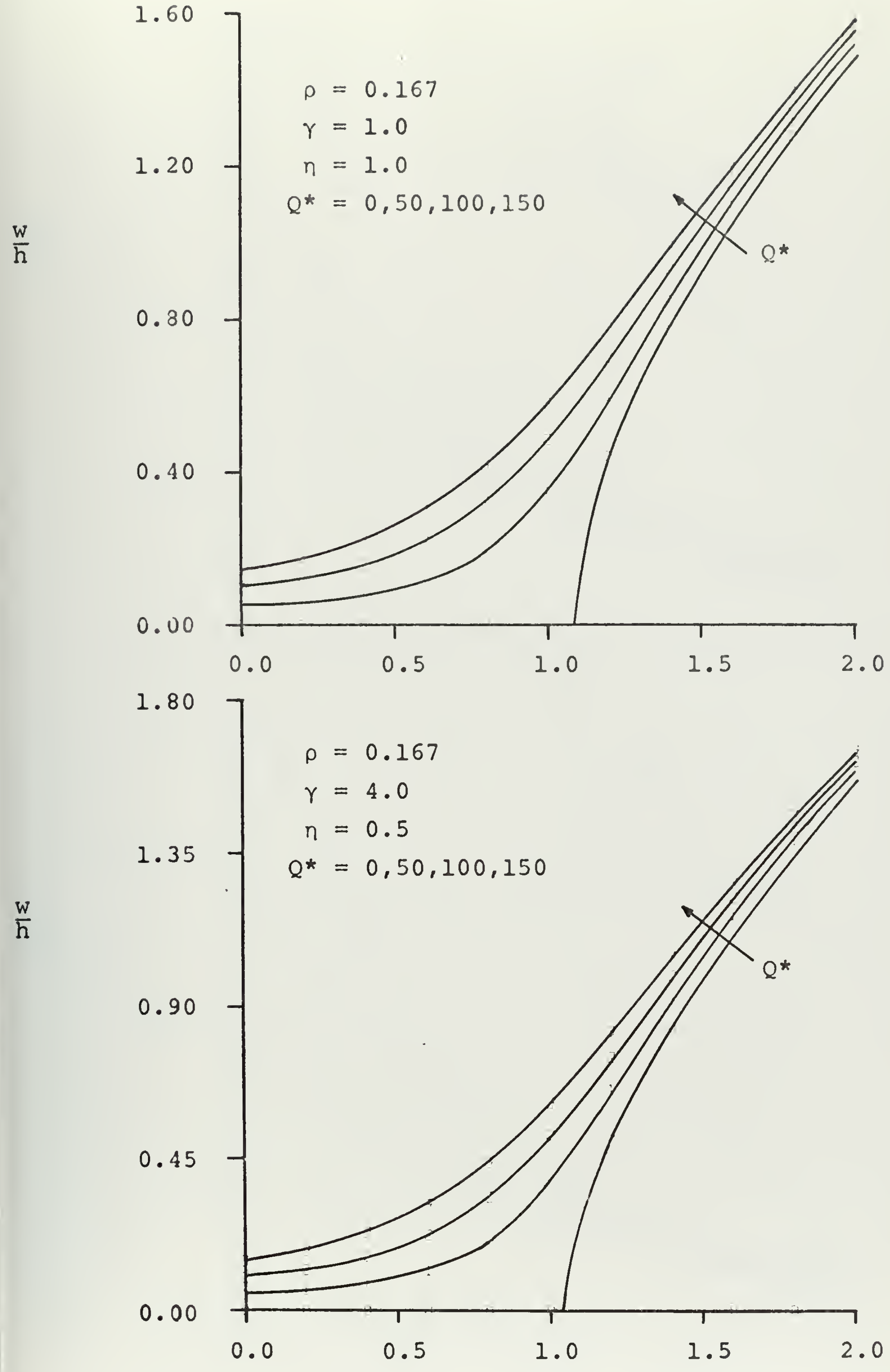
Figures 14 & 15



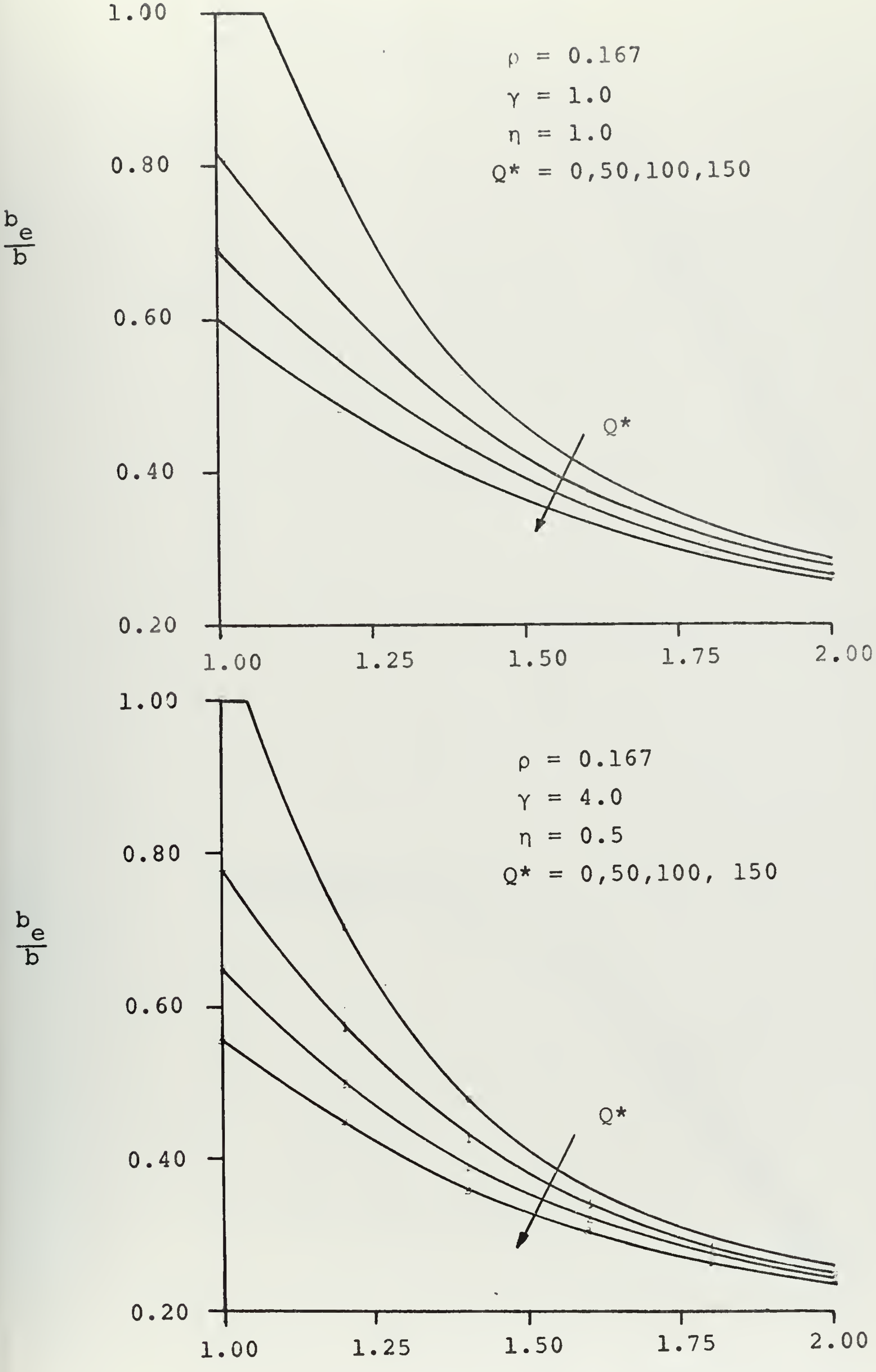
Figures 16 & 17



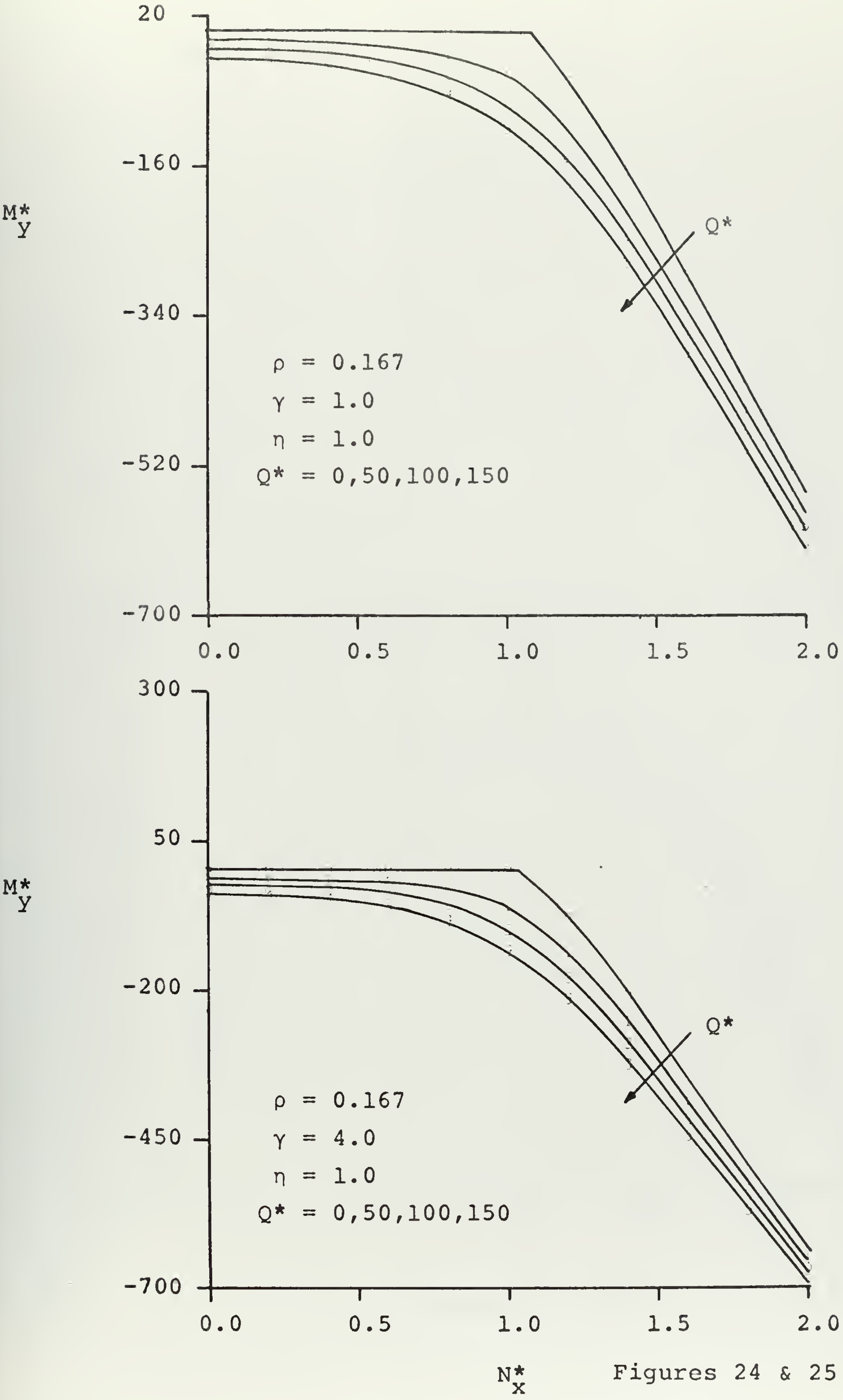
Figures 18 & 19



Figures 20 & 21

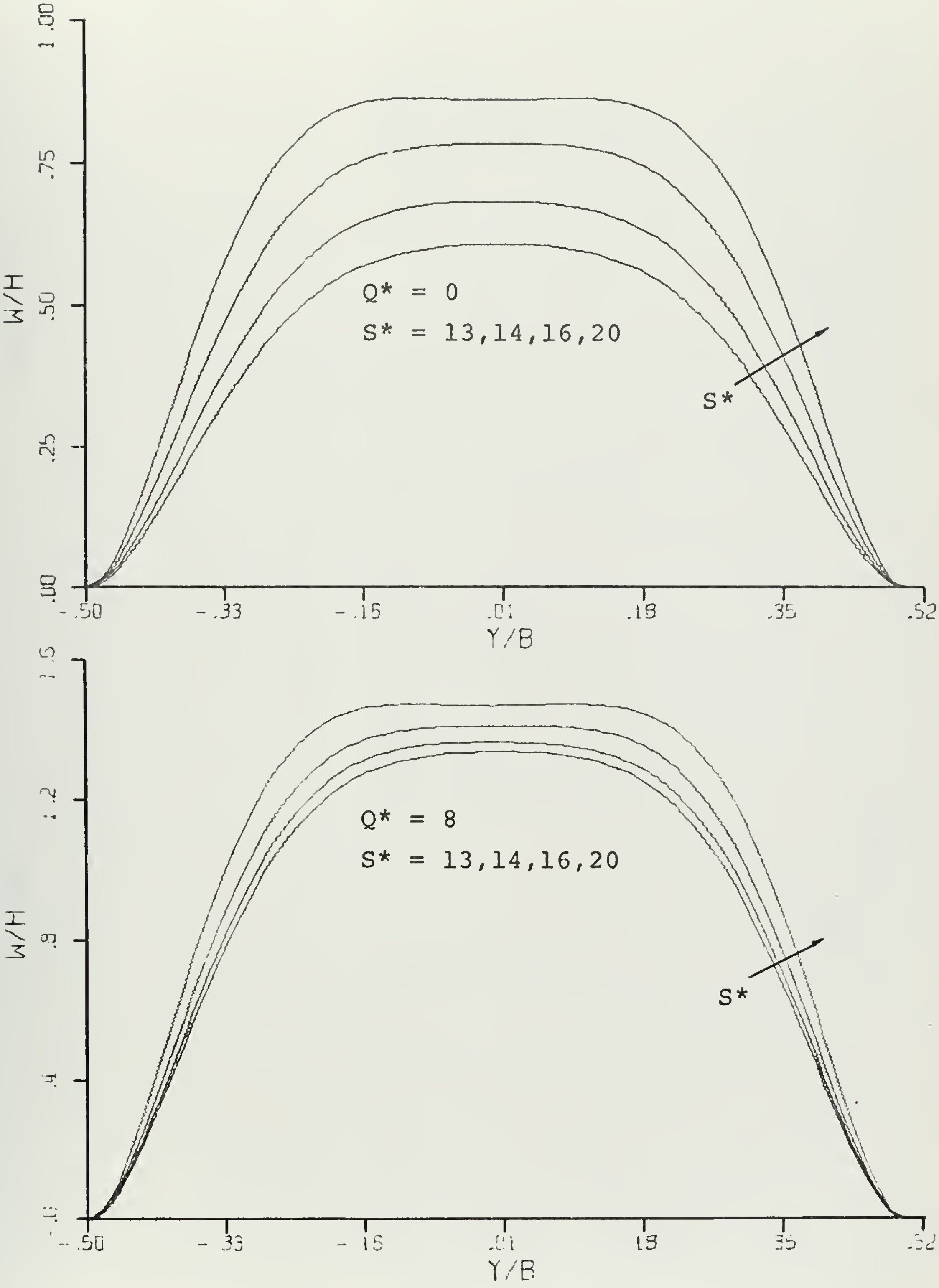


Figures 22 & 23



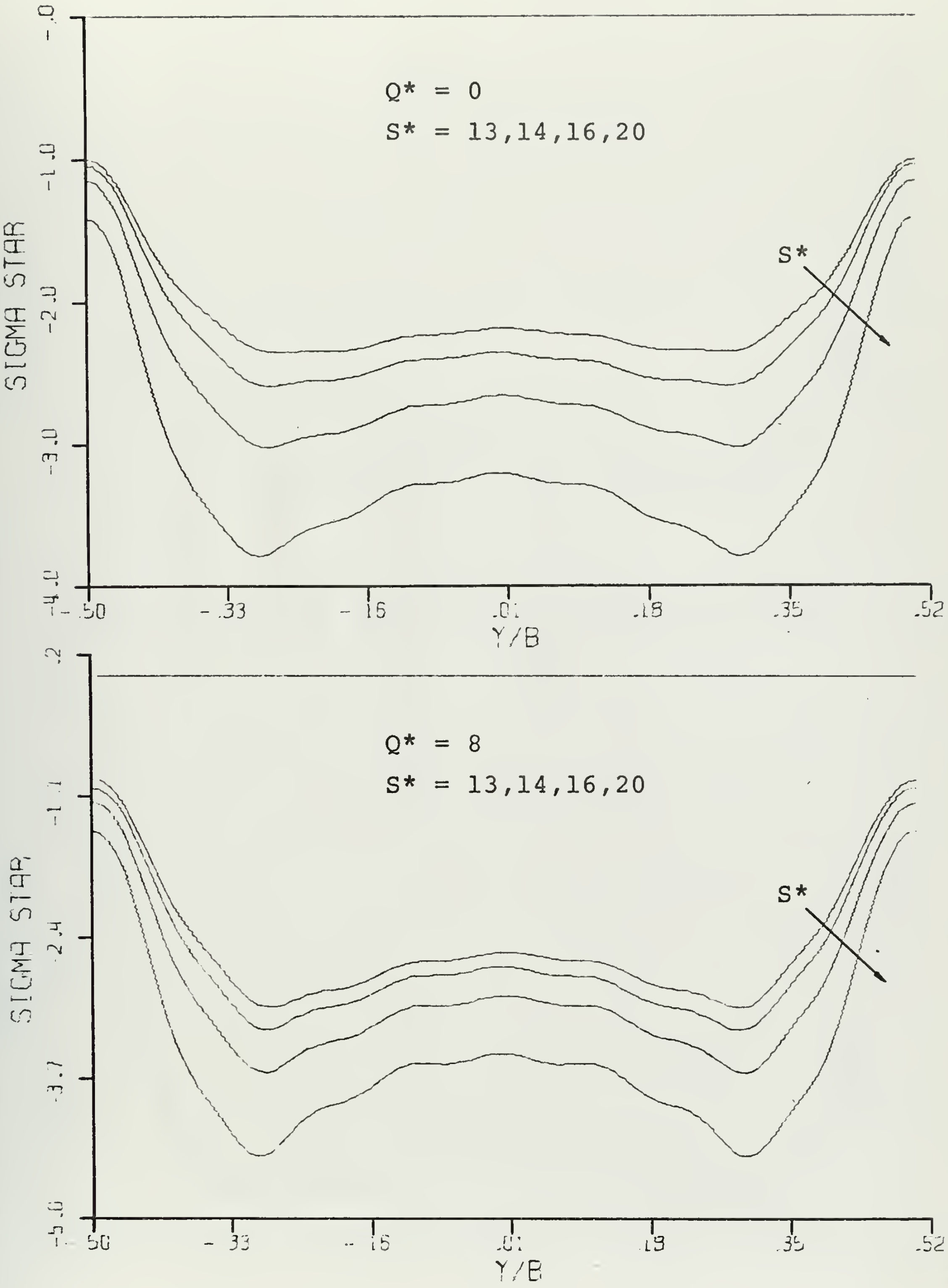
Figures 24 & 25

ρ = 0.667 γ = η = 1.0 N* = 0



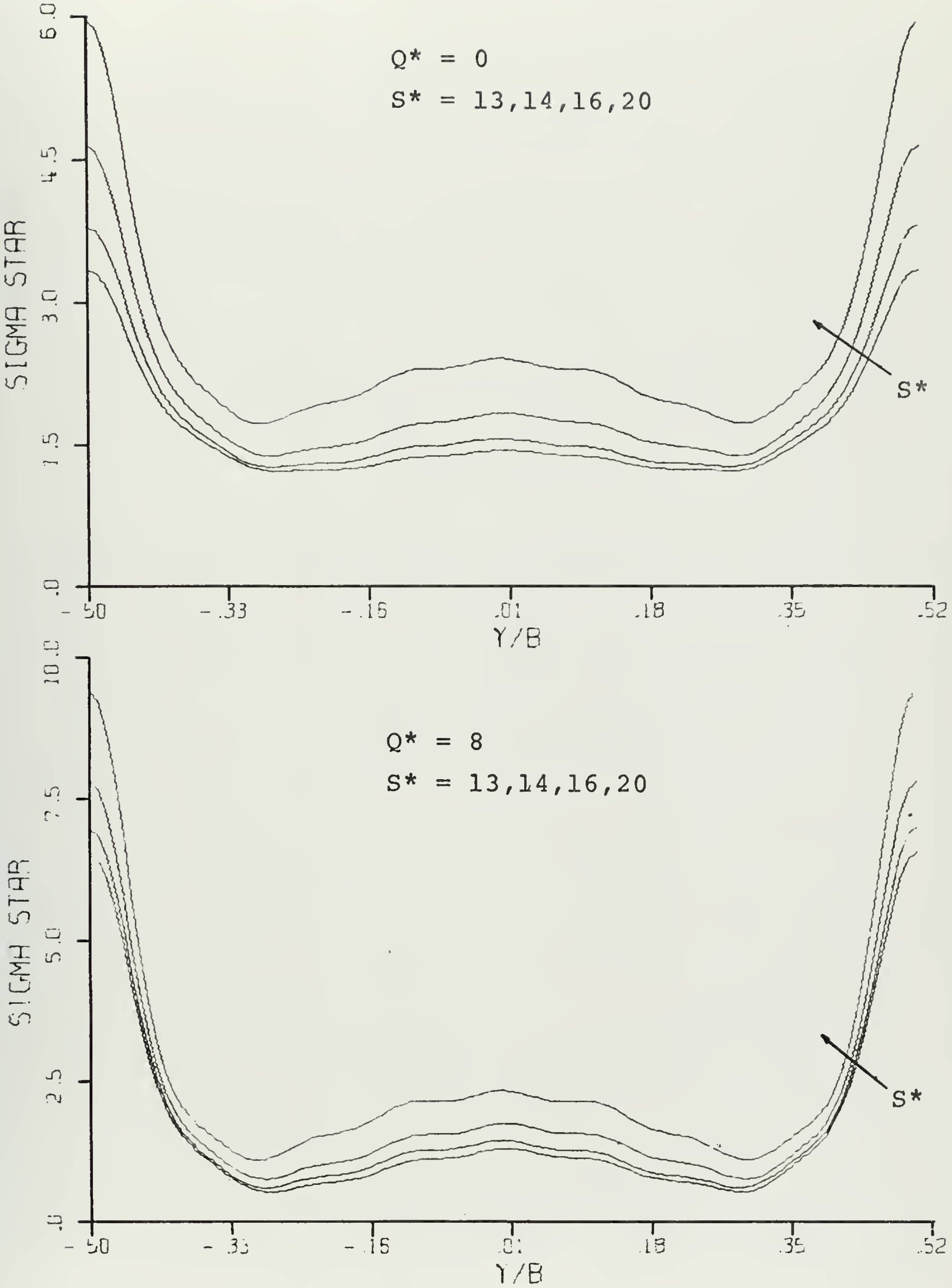
Figures 26 & 27

ρ = 0.667 γ = η = 1.0 N* = 0



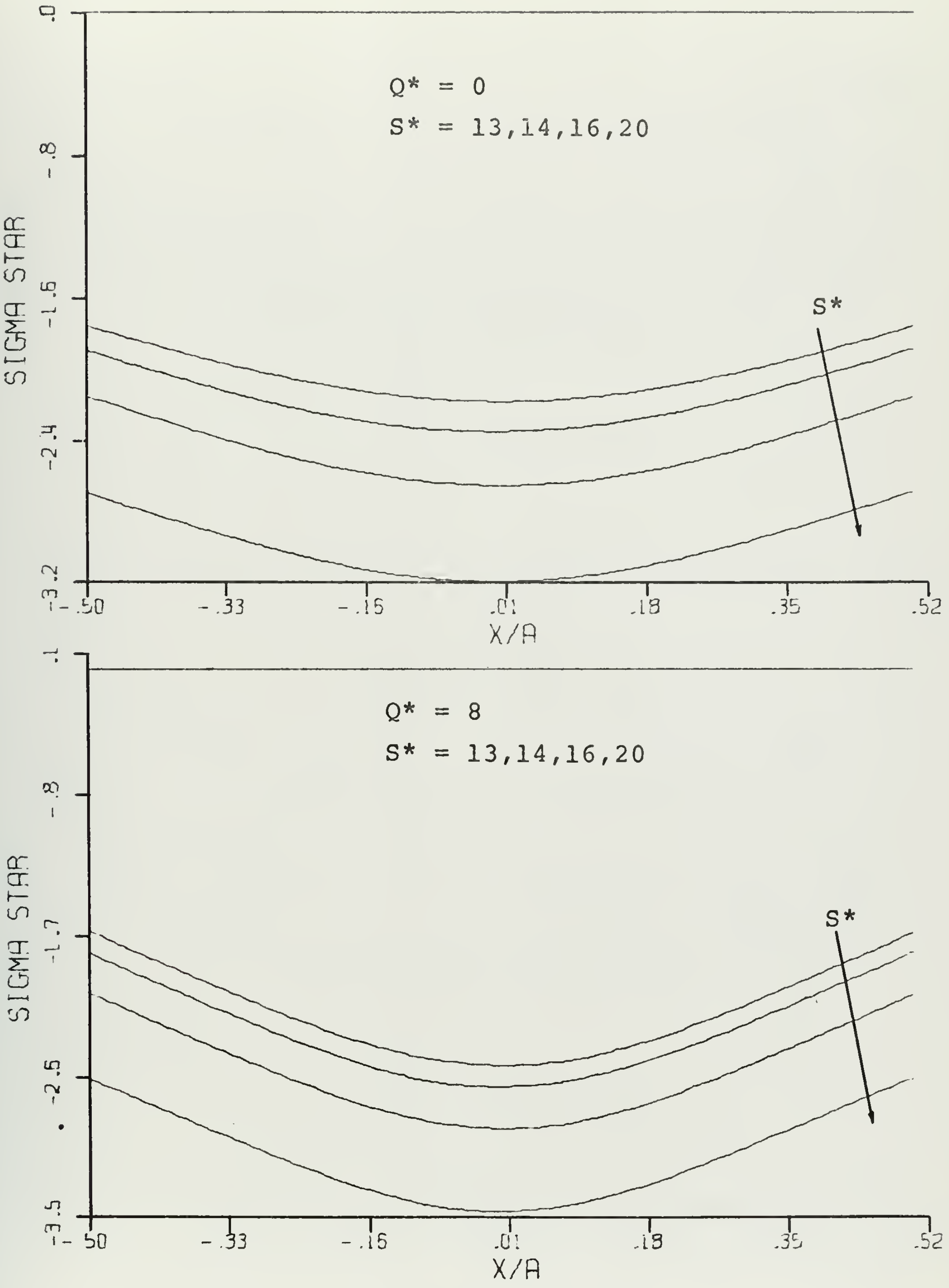
Figures 28 & 29

ρ = 0.667 γ = η = 1.0 N* = 0



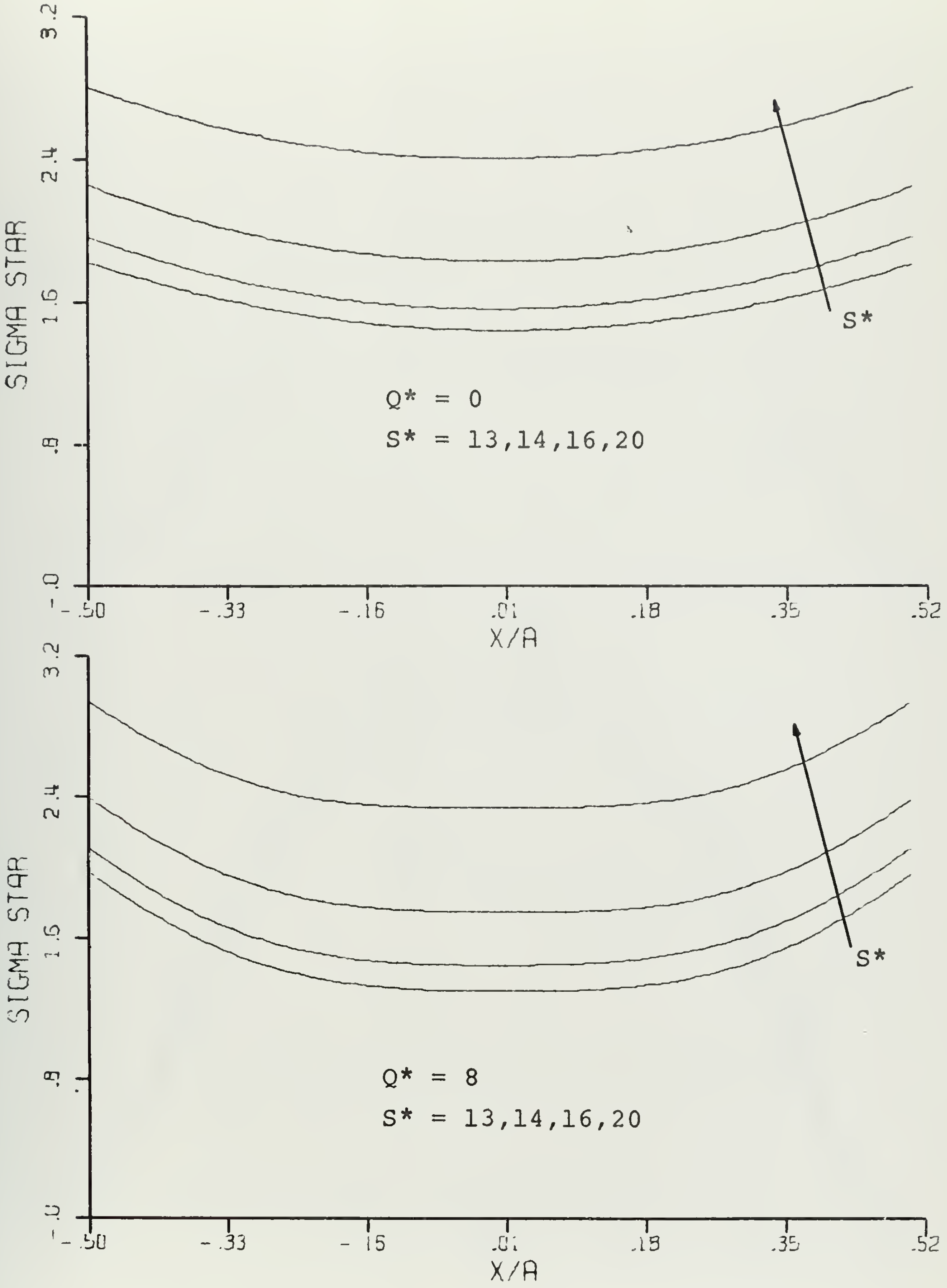
Figures 30 & 31

$\rho = 0.667$ $\gamma = \eta = 1.0$ $N^* = 0$



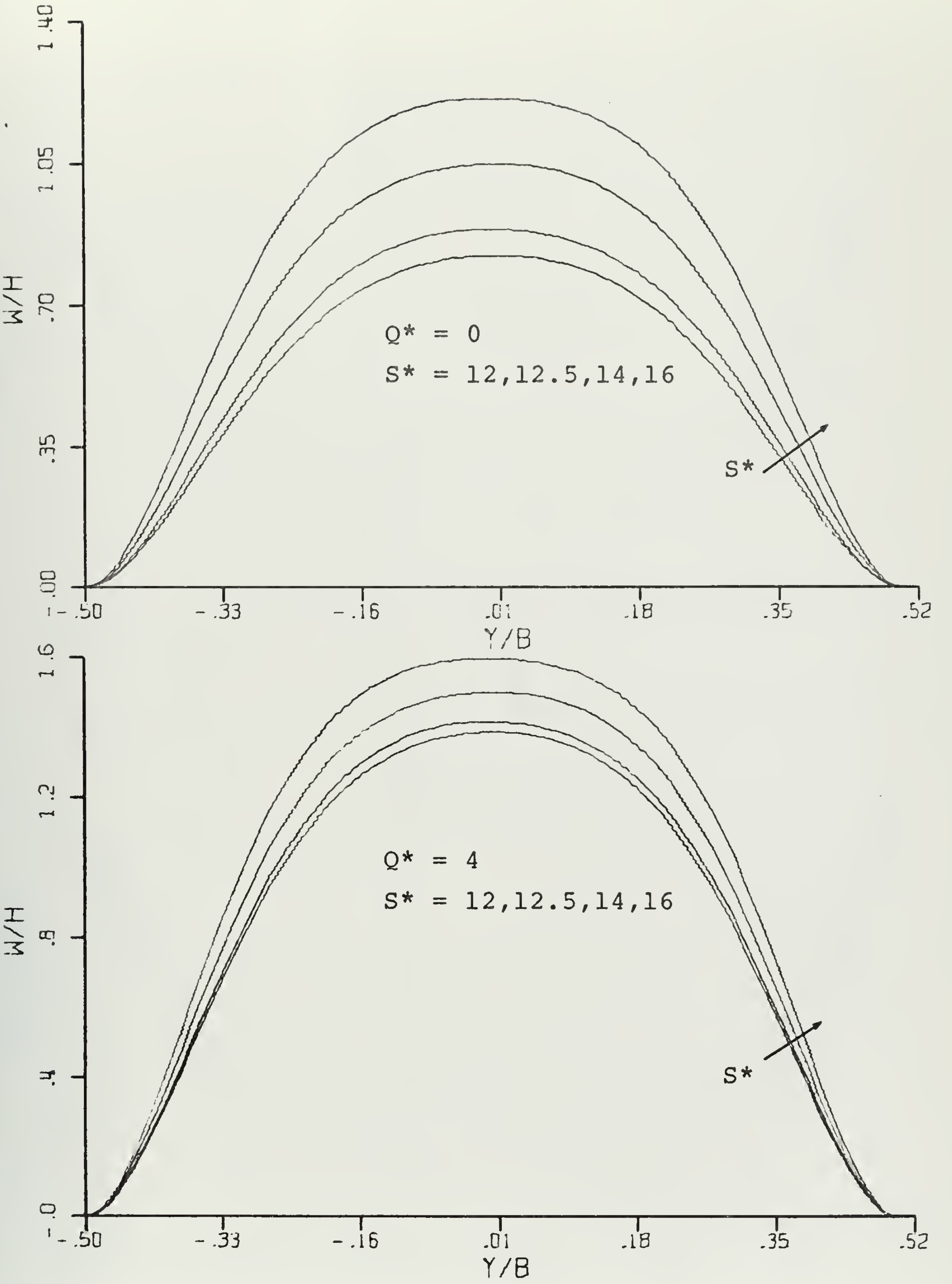
Figures 32 & 33

$\rho = 0.667 \qquad \gamma = \eta = 1.0 \qquad N^* = 0$



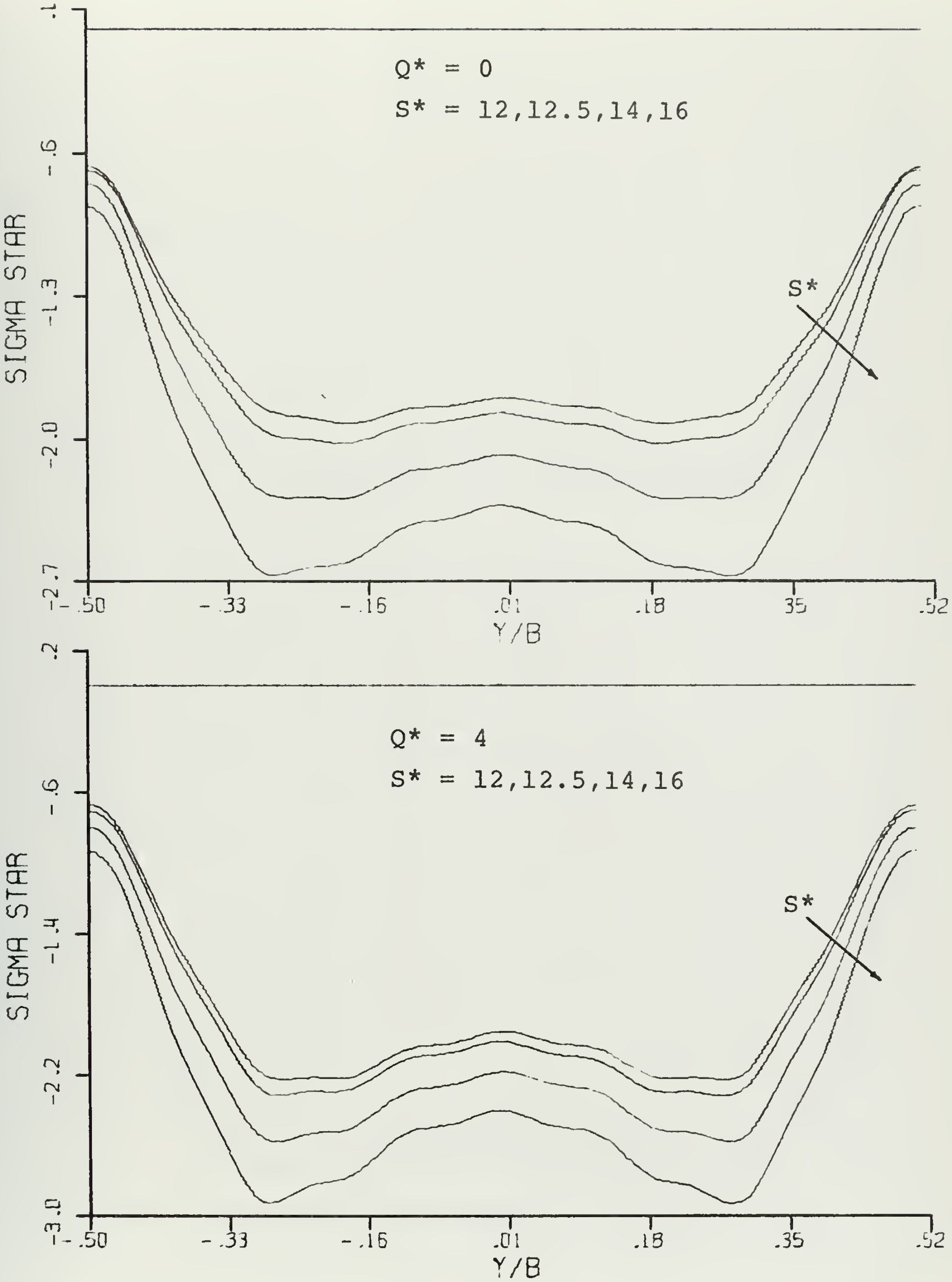
Figures 34 & 35

ρ = 0.80 $\gamma = \eta$ = 1.0 N* = 0



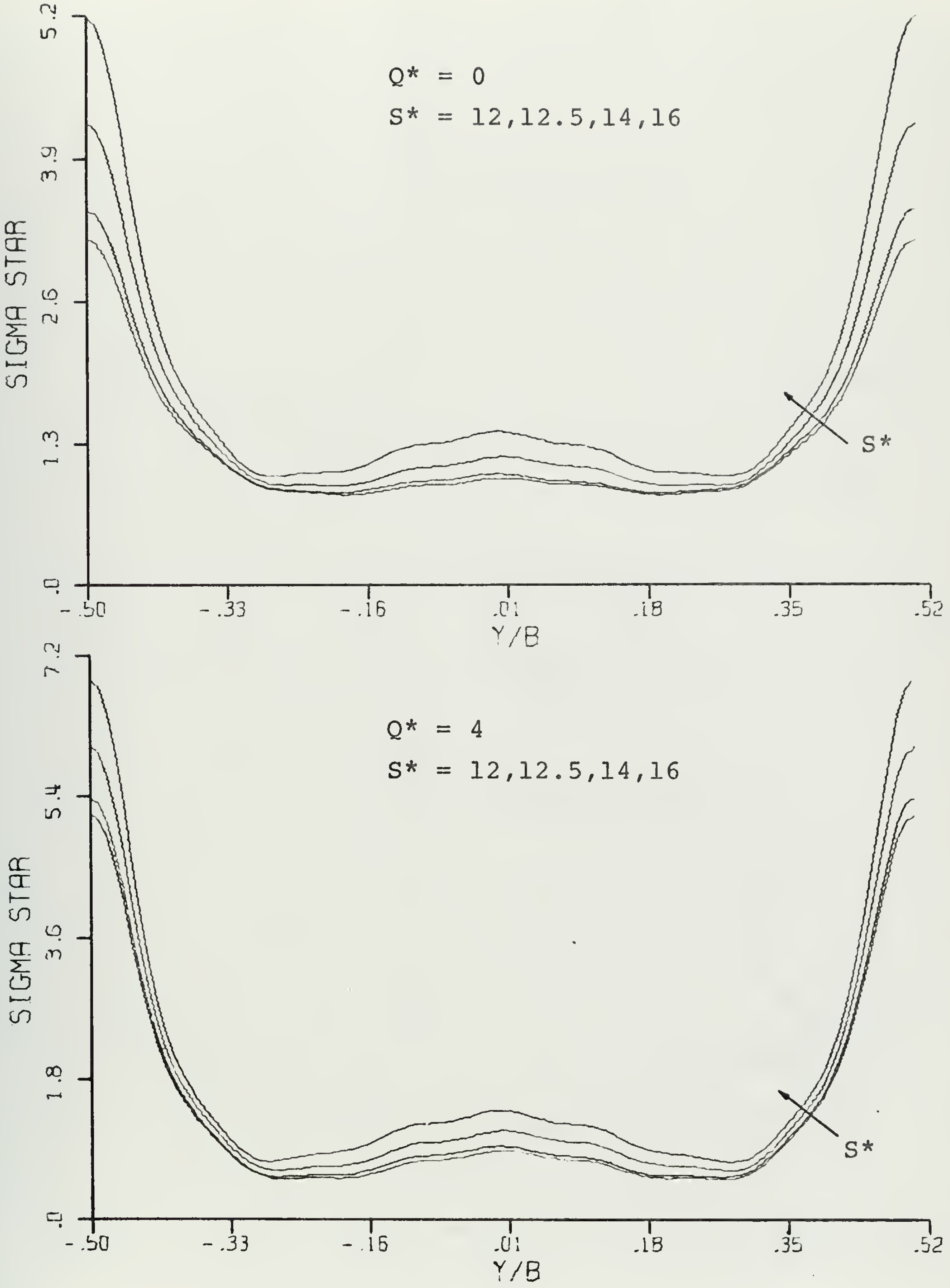
Figures 36 & 37

$\rho = 0.80 \qquad \gamma = \eta = 1.0 \qquad N^* = 0$



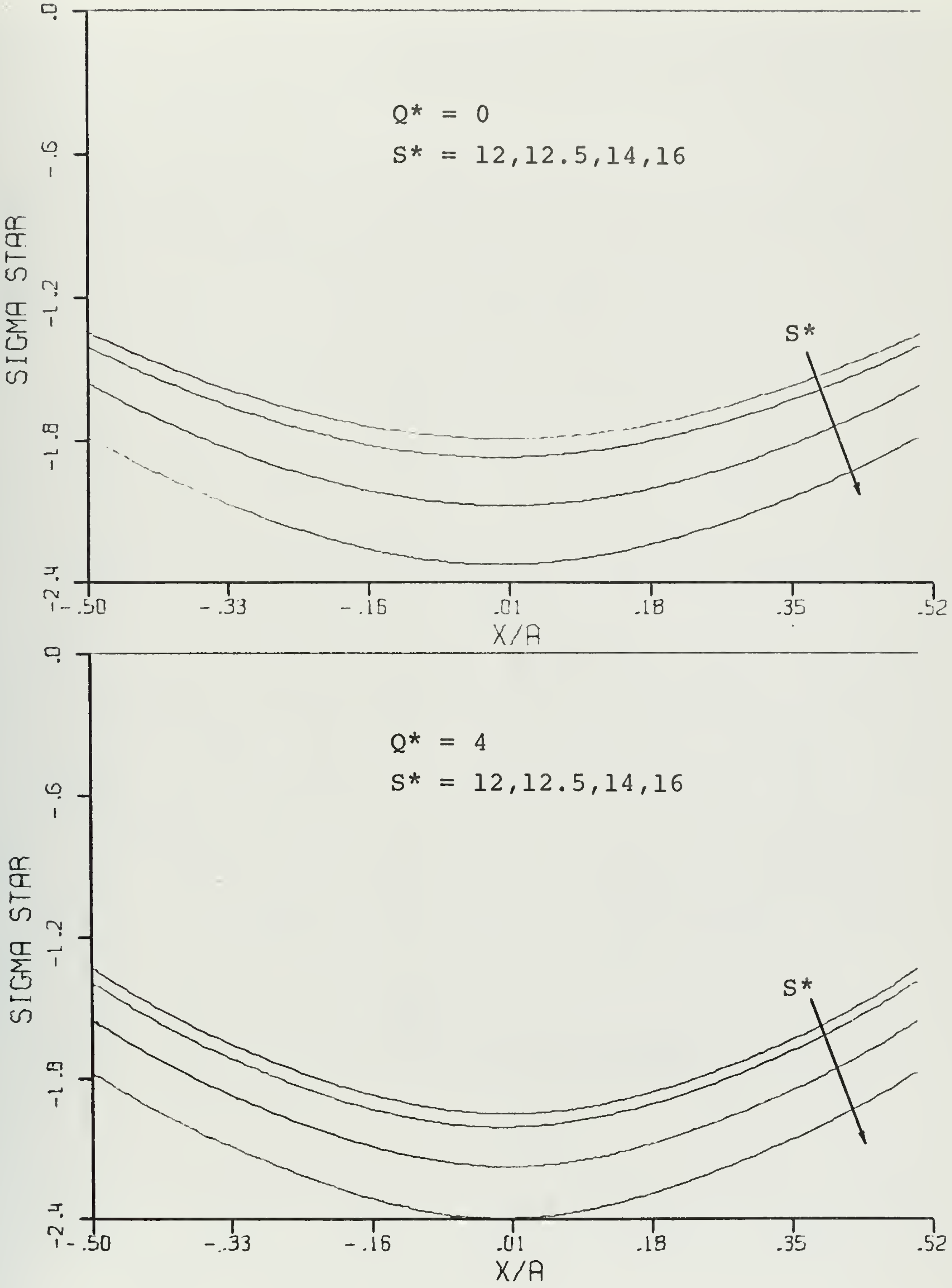
Figures 38 & 39

$\rho = 0.80$ $\gamma = \eta = 1.0$ $N^* = 0$



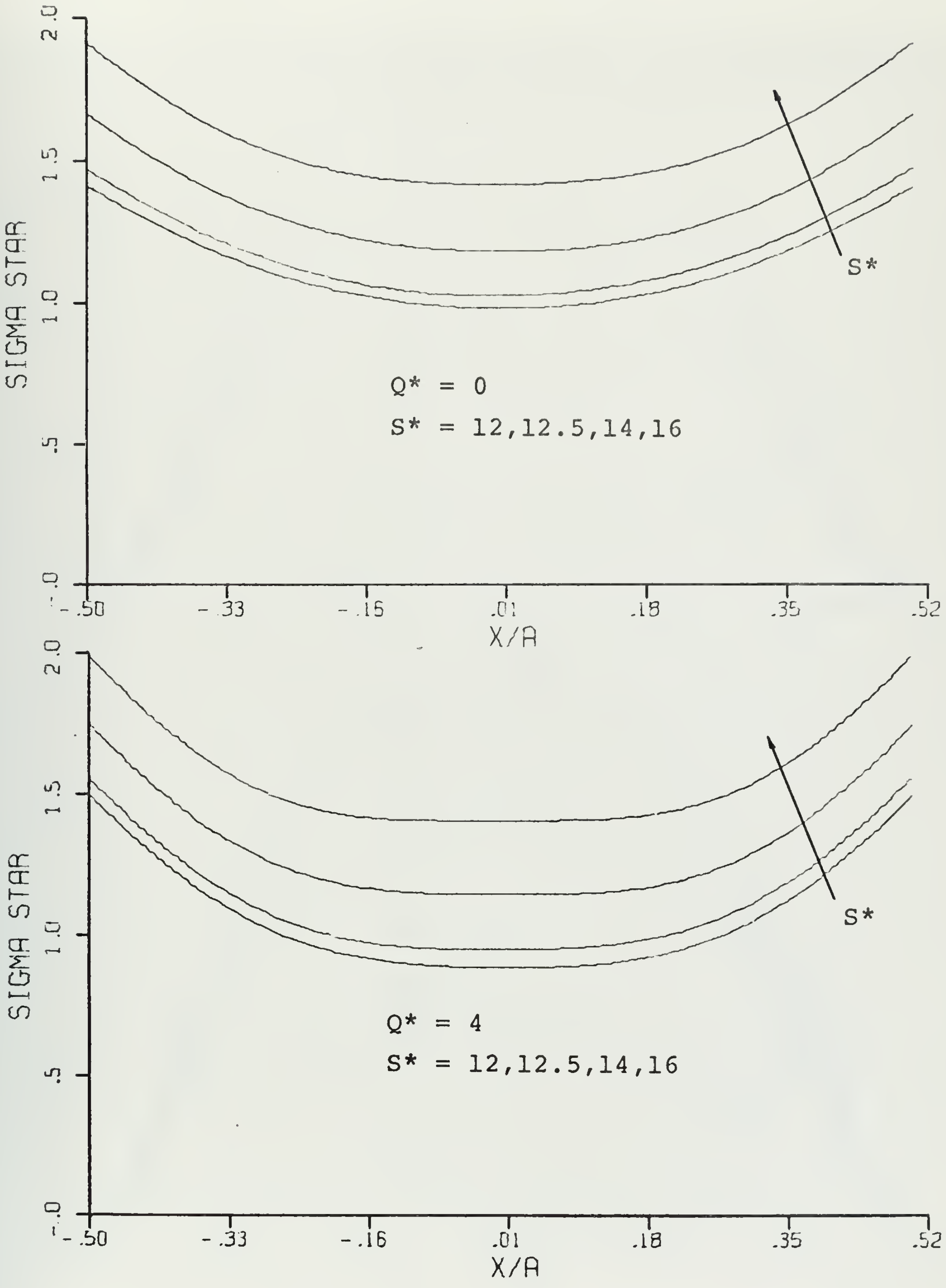
Figures 40 & 41

$\rho = 0.80 \qquad \gamma = \eta = 1.0 \qquad N^* = 0$



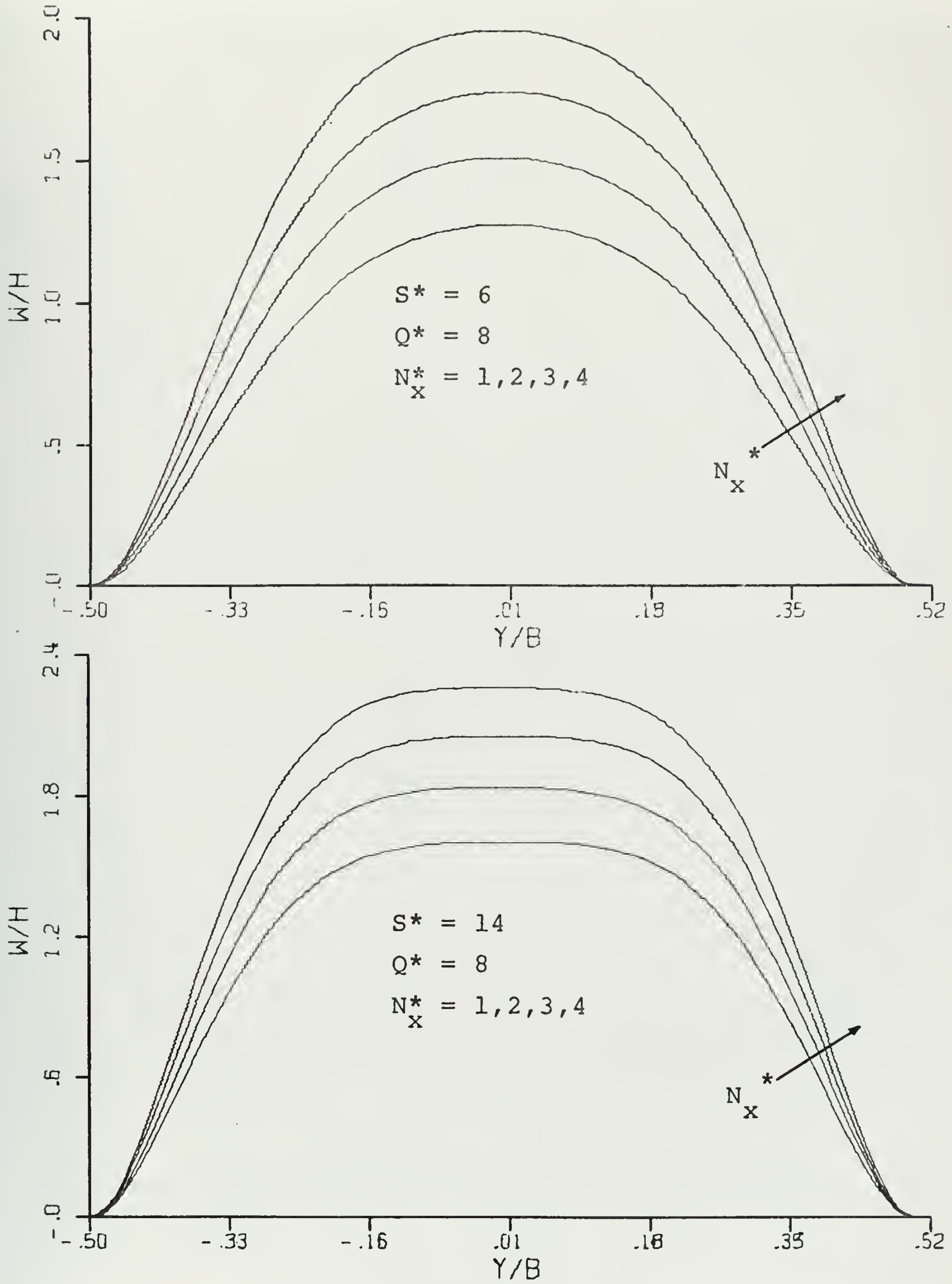
Figures 42 & 43

$\rho = 0.80 \qquad \gamma = \eta = 1.0 \qquad N^* = 0$



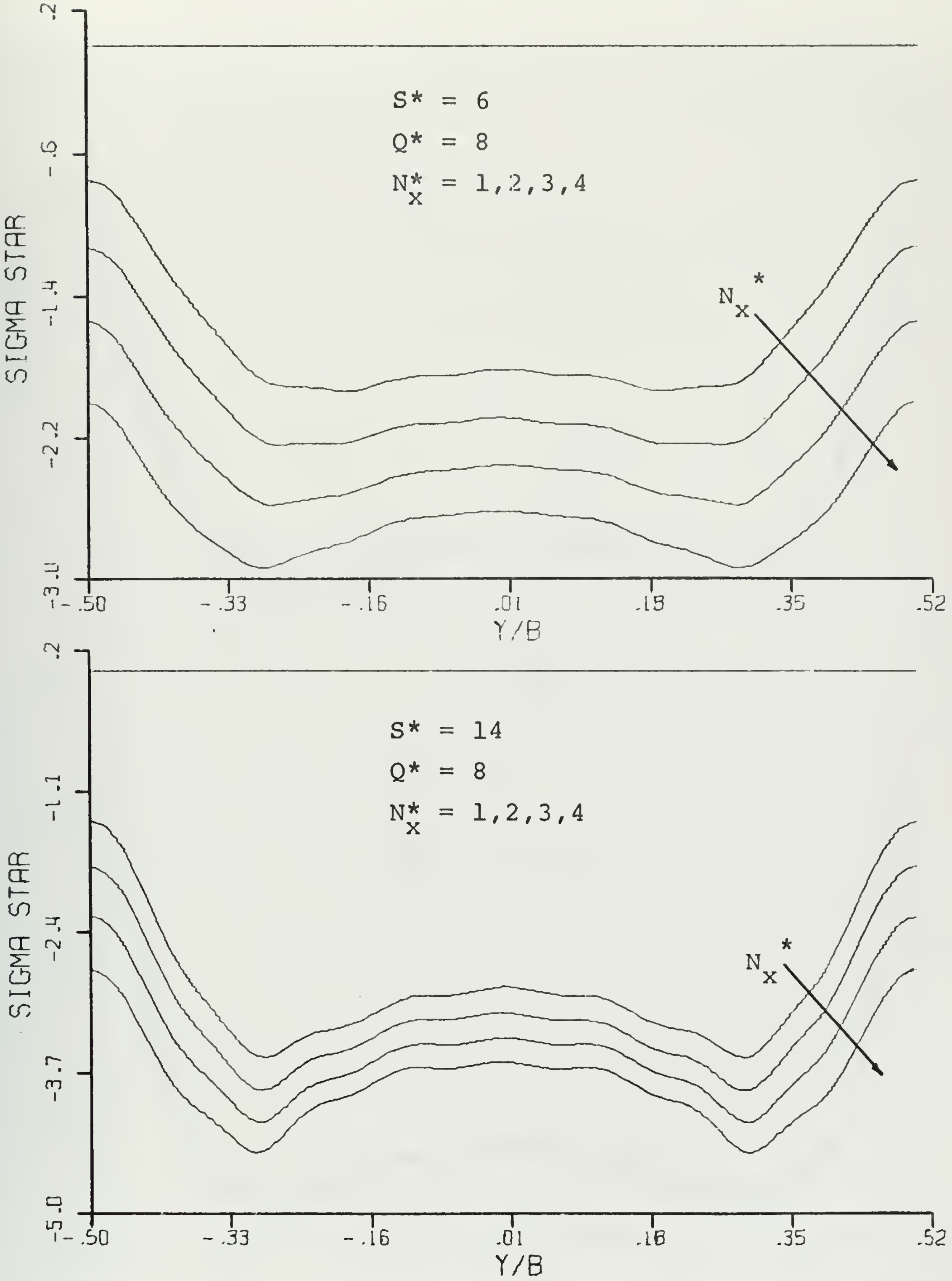
Figures 44 & 45

$\rho = 0.667 \qquad \gamma = \eta = 1.0 \qquad N_y^* = 0$



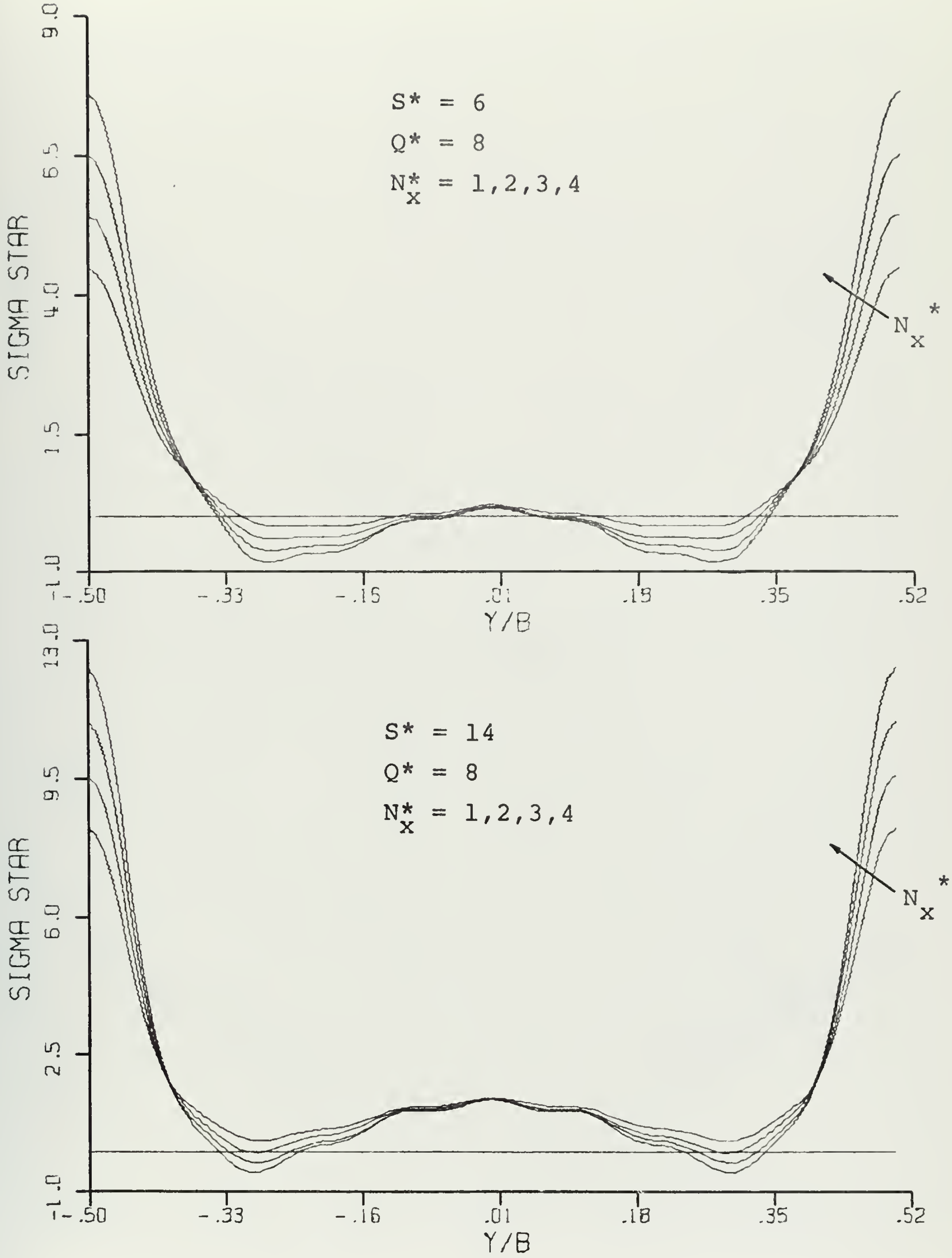
Figures 46 & 47

$\rho = 0.667 \quad \gamma = \eta = 1.0 \quad N_y^* = 0$



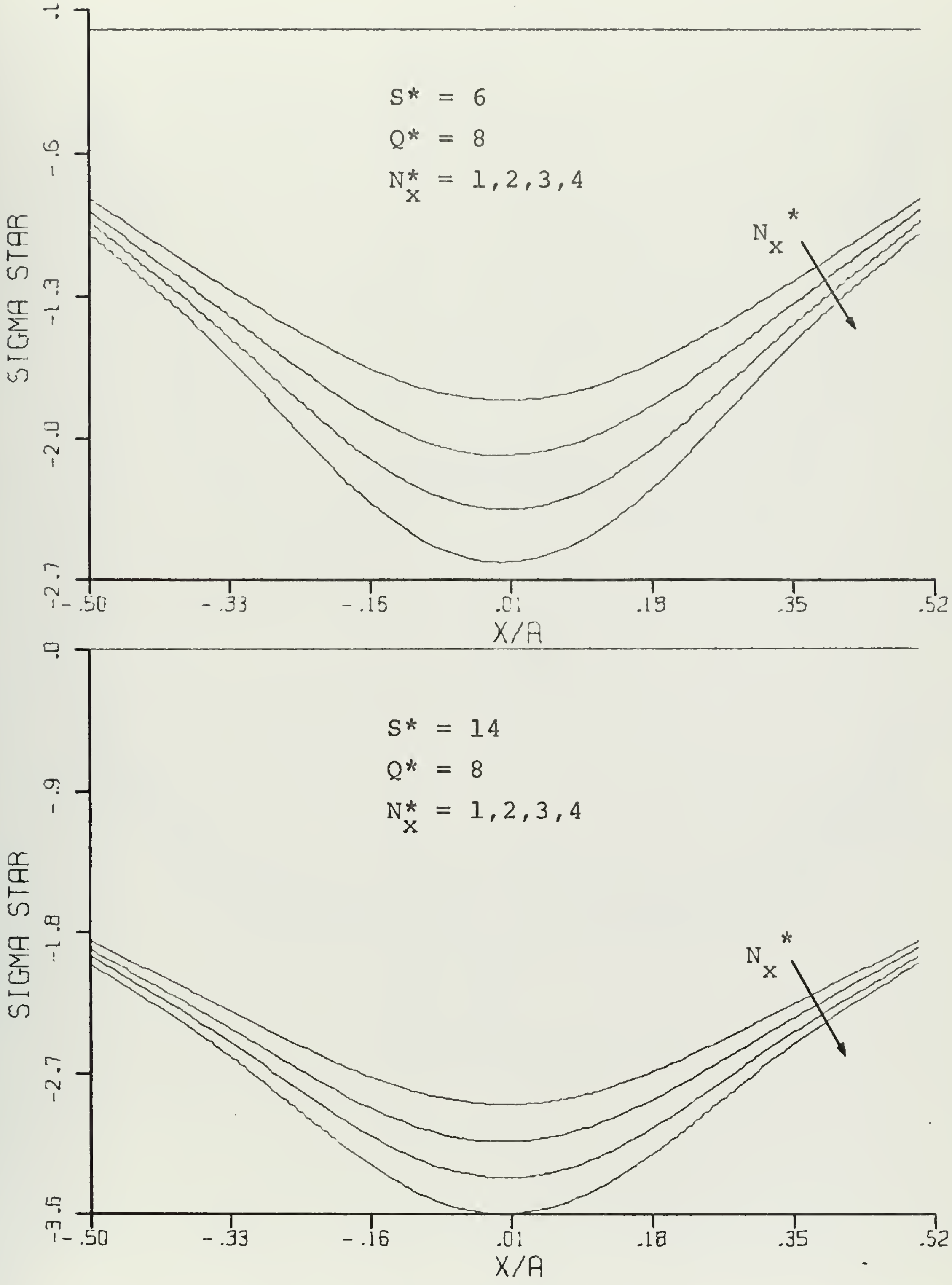
Figures 48 & 49

$\rho = 0.667 \qquad \gamma = \eta = 1.0 \qquad N_Y^* = 0$



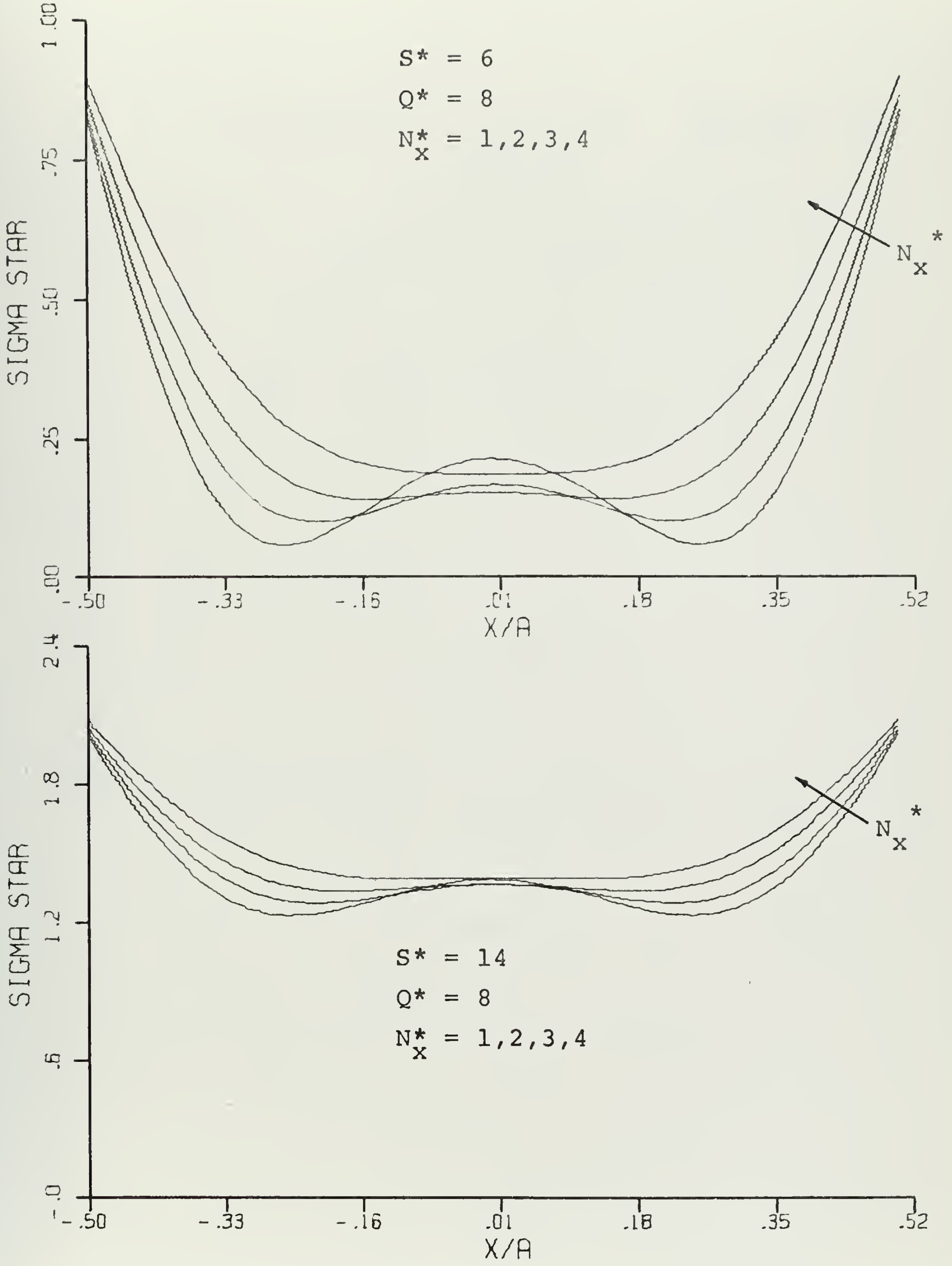
Figures 50 & 51

$\rho = 0.667 \quad \gamma = \eta = 1.0 \quad N_y^* = 0$



Figures 52 & 53

$\rho = 0.667 \qquad \gamma = \eta = 1.0 \qquad N_y^* = 0$



Figures 54 & 55

$\rho = 0.667 \qquad \gamma = \eta = 1.0 \qquad Q^* = N^* = 0$

$S^* = 14$

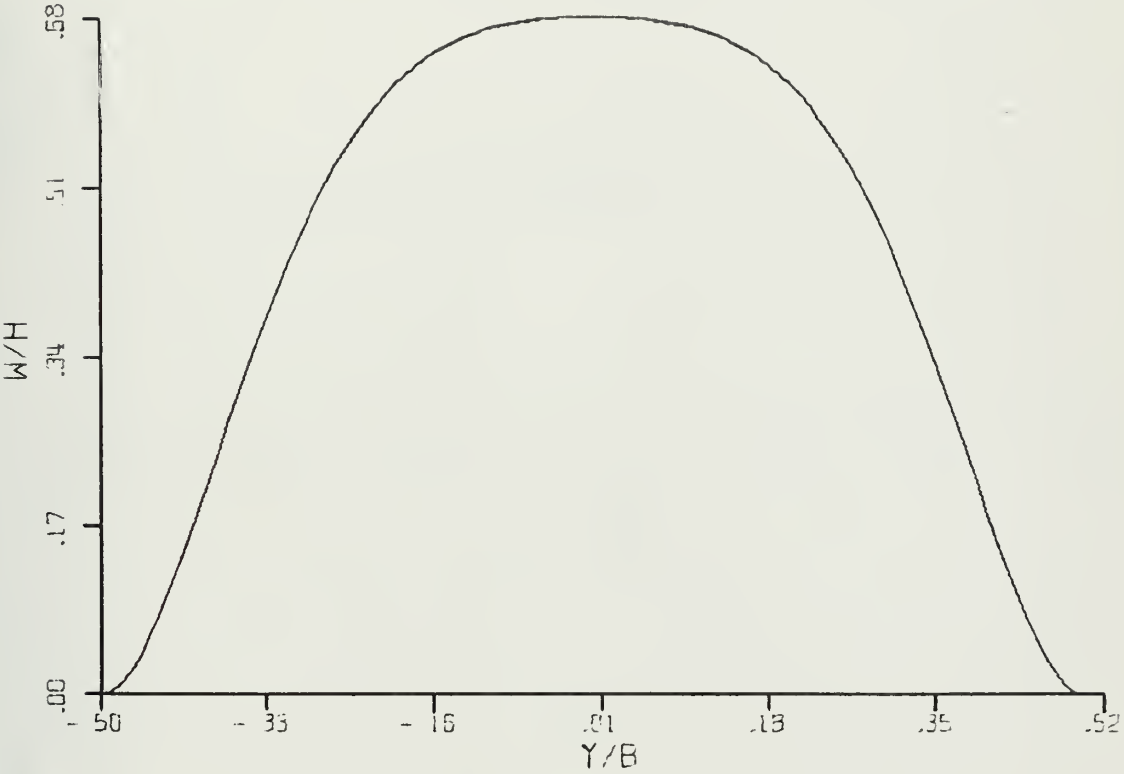
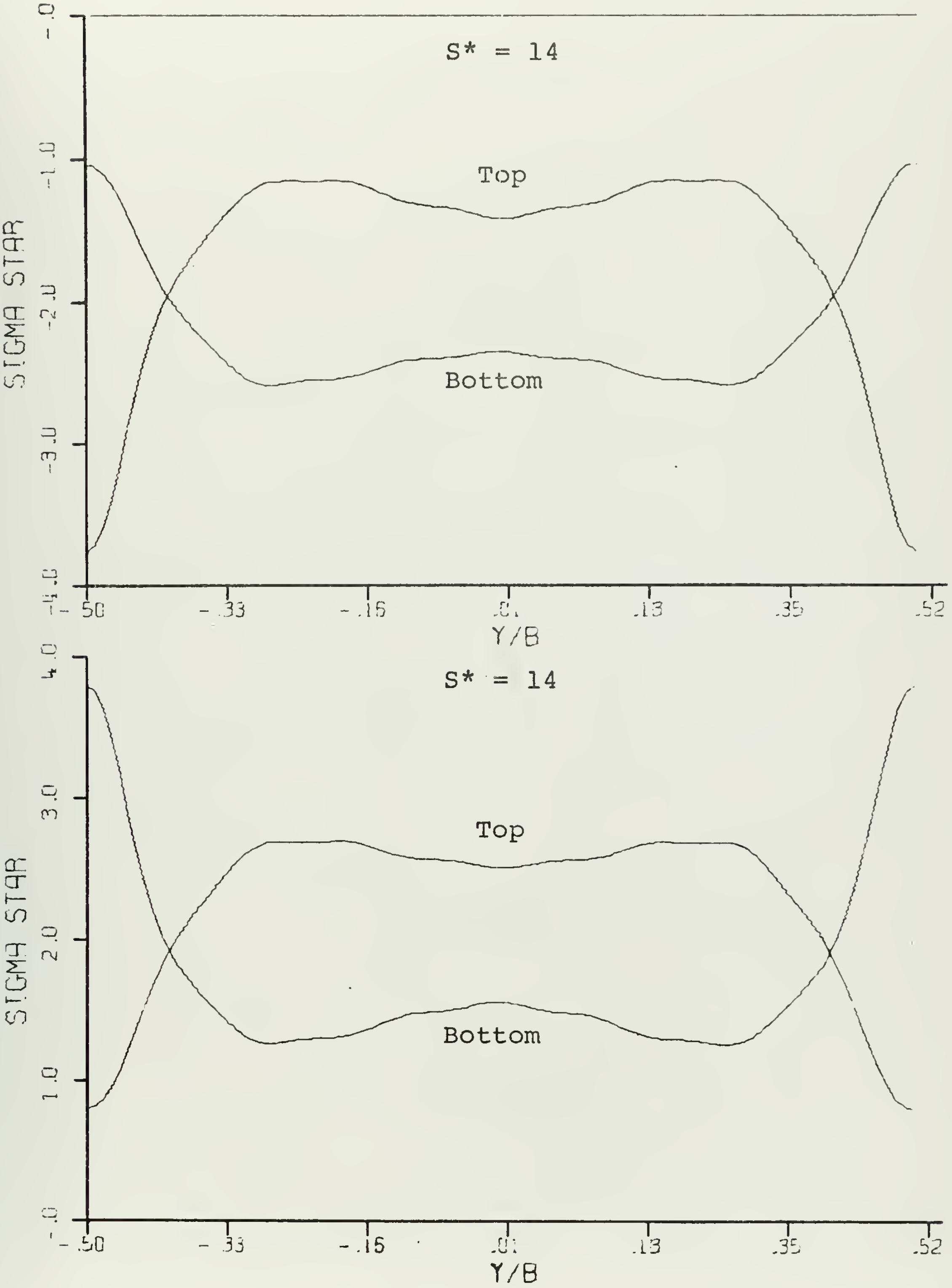


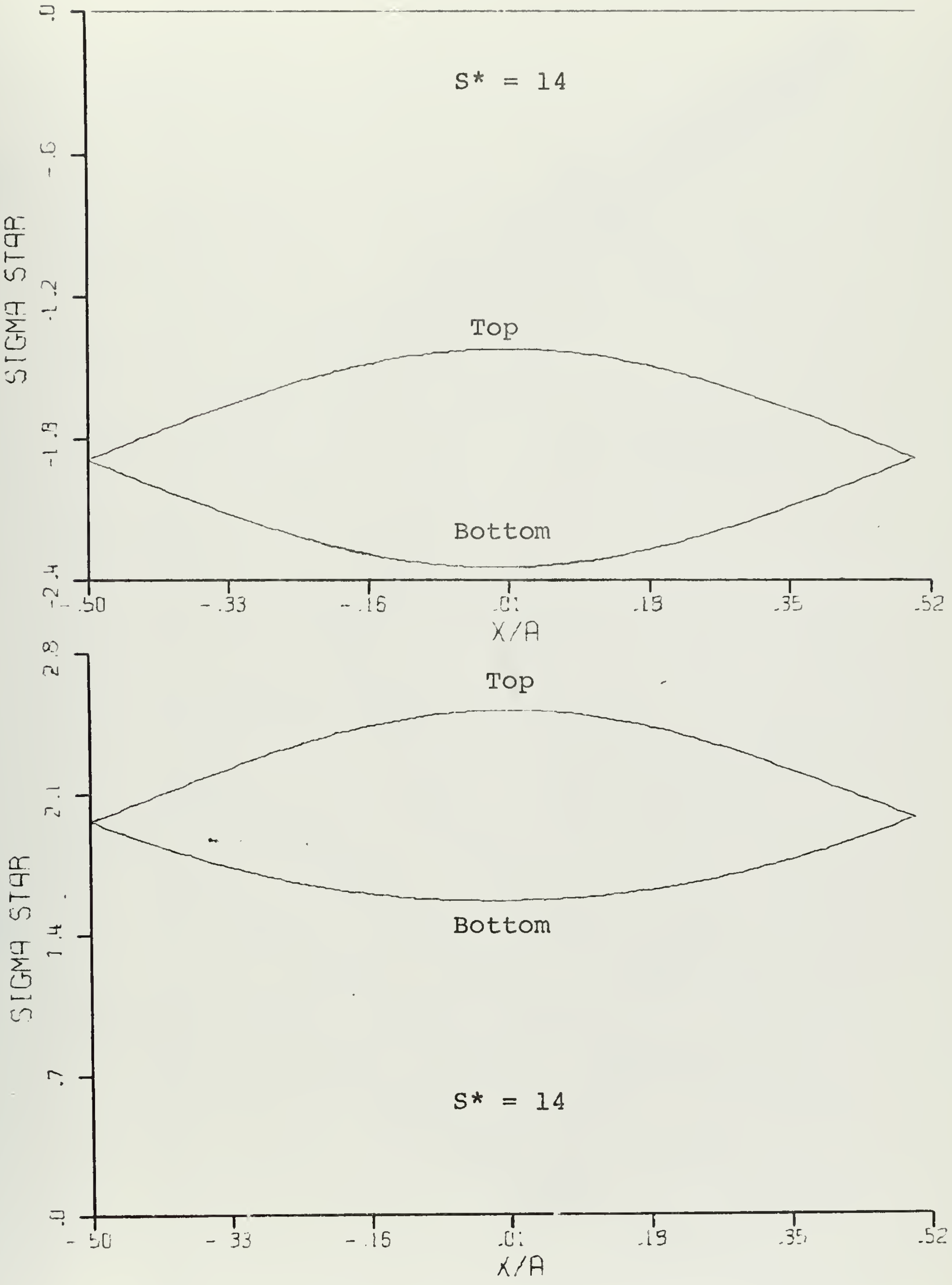
Figure 56

$\rho = 0.667 \qquad \gamma = \eta = 1.0 \qquad Q^* = N^* = 0$

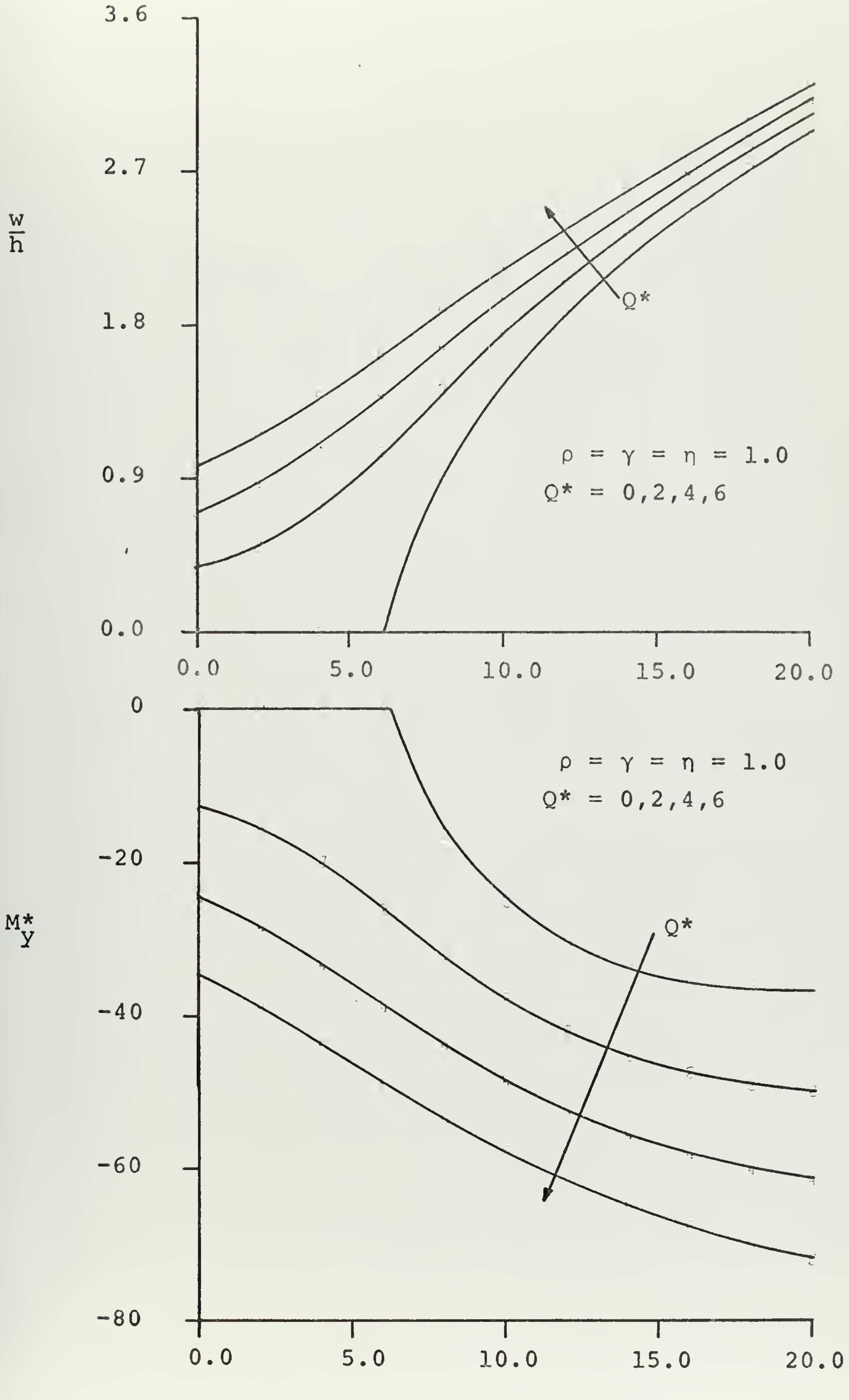


Figures 57 & 58

$\rho = 0.667$ $\gamma = \eta = 1.0$ $Q^* = N^* = 0$



Figures 59 & 60



Figures 61 & 62

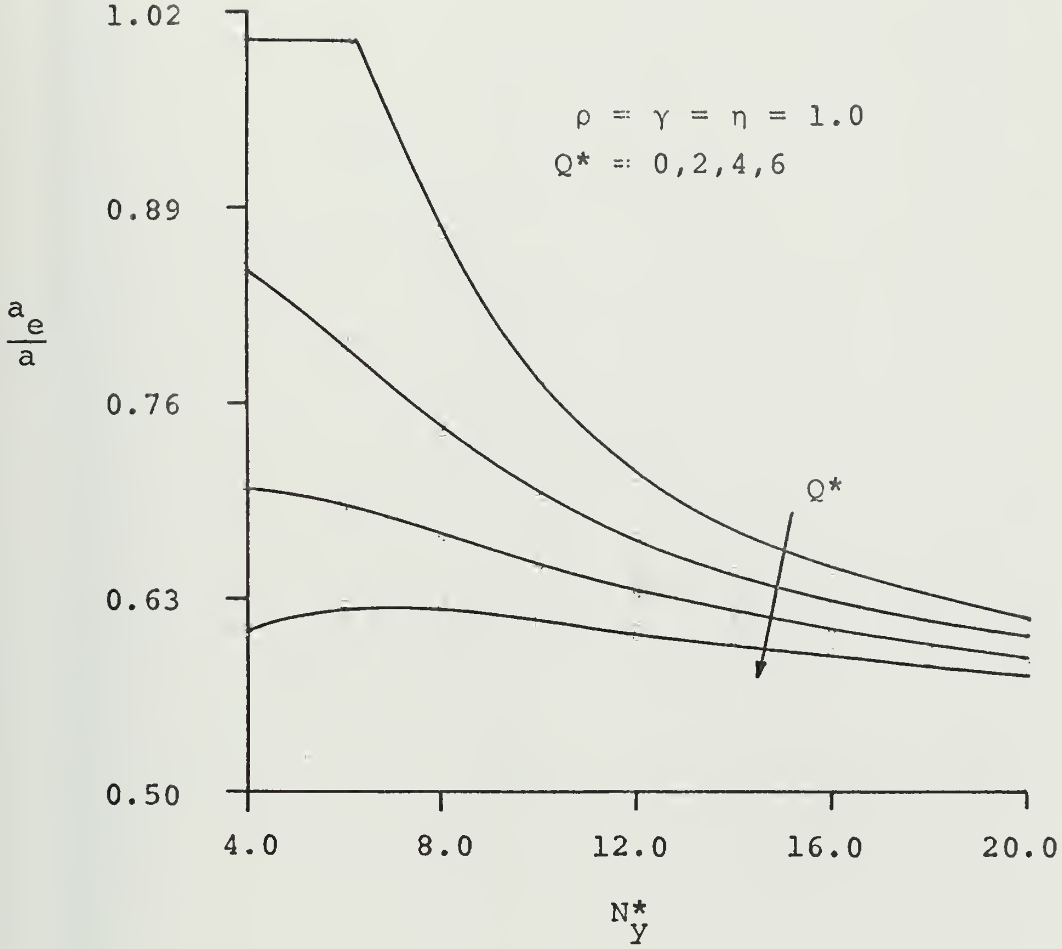


Figure 63

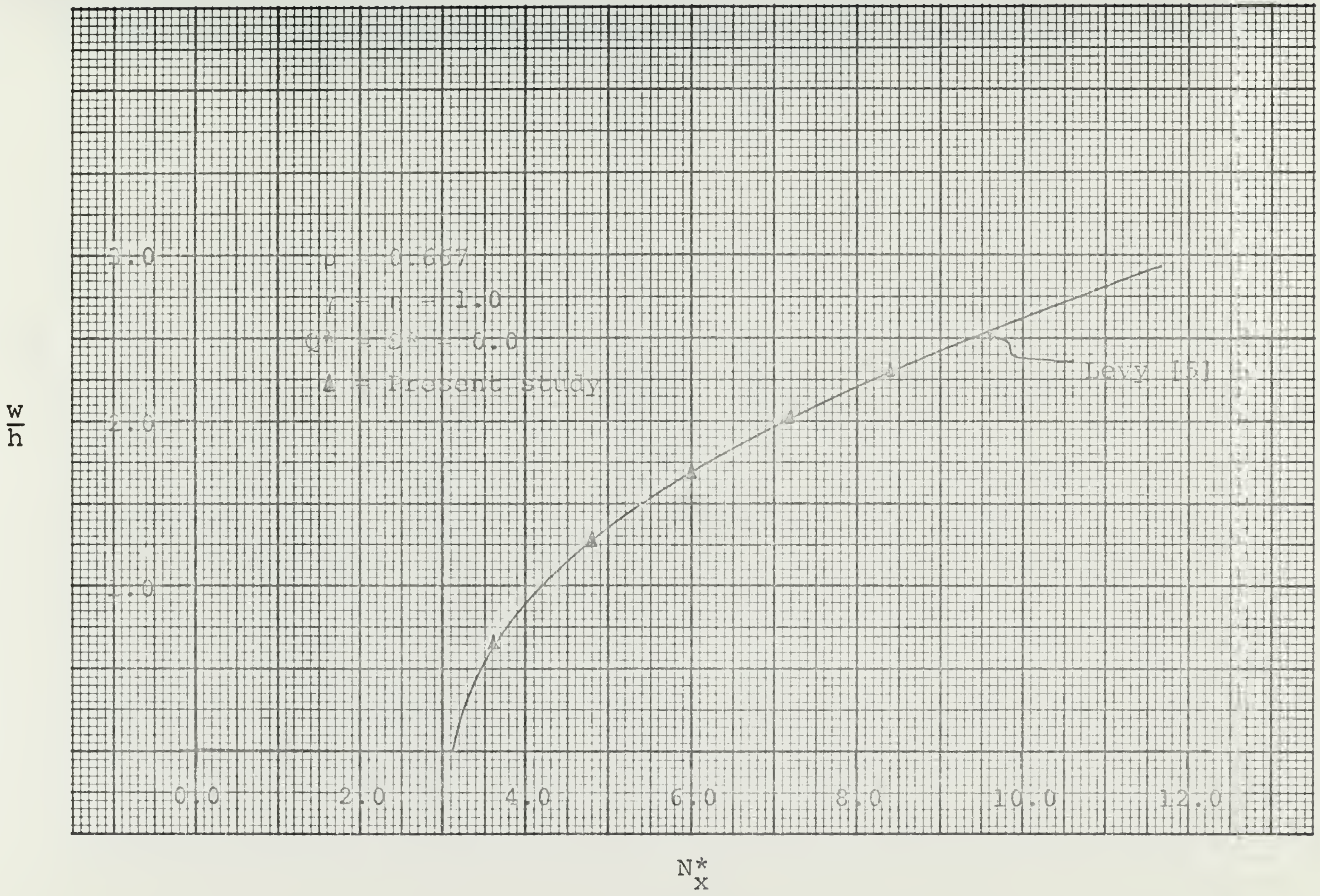

$\frac{w}{h}$
3.0
2.0
1.0
0.0
2.0
4.0
6.0
8.0
10.0
12.0
N_x^*
$\rho = 0.667$
$\gamma = \eta = 1.0$
$Q^* = S^* = 0.0$
▲ = Present Study
Levy [5]

Figure 64

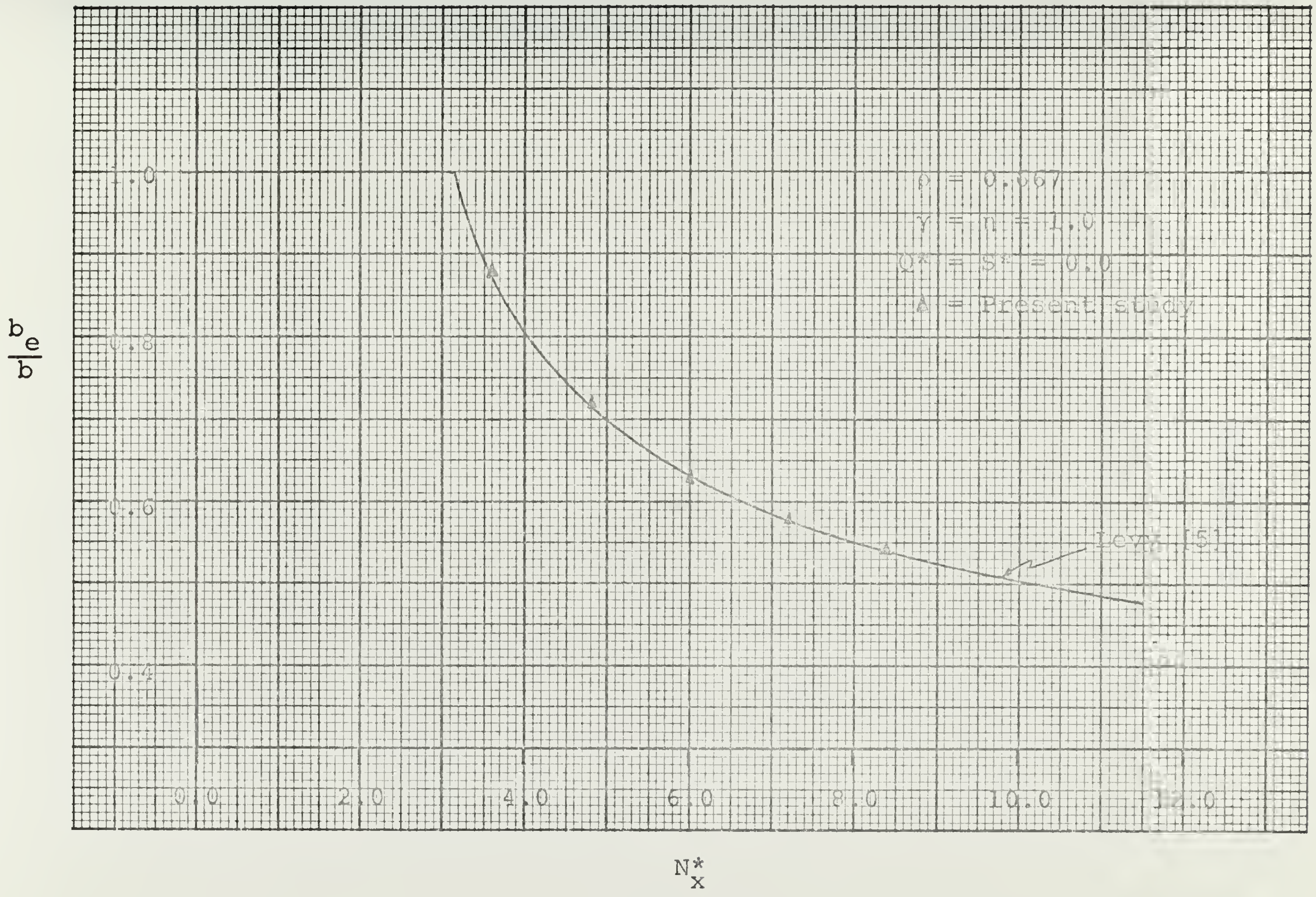

$\frac{b_e}{b}$
1.0
0.8
0.6
0.4
0.0
2.0
4.0
6.0
8.0
10.0
12.0
N_x^*
ρ = 0.667
γ = n = 1.0
Q* = S* = 0.0
▲ = Present study
Levy [5]

Figure 65

9. Appendices

APPENDIX A

Details of Derivation of Basic Equations of Large Deflections

Definition of External Forces and Moments

Shearing force per unit length parallel to x axis

$$Q_x = \int_{-h/2}^{h/2} \tau_{xz} dz$$

Shearing force per unit length parallel to y axis

$$Q_y = \int_{-h/2}^{h/2} \tau_{yz} dz$$

Bending moments per unit length of sections of a plate perpendicular to x and y axis respectively

$$M_x = \int_{-h/2}^{h/2} z \sigma_x dz$$

$$M_y = \int_{-h/2}^{h/2} z \sigma_y dz$$

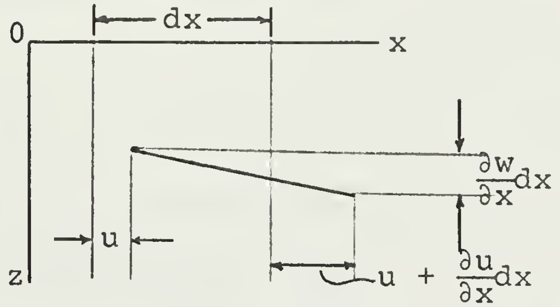
Twisting moment per unit length of section of a plate perpendicular to x and y axis respectively

$$M_{xy} = -\int_{-h/2}^{h/2} z \tau_{xy} dz$$

$$M_{yx} = -\int_{-h/2}^{h/2} z \tau_{yx} dz$$

Strain - Displacement Relations

Consideration of strain in the middle of the plate


0
dx
x
z
u
$\frac{\partial w}{\partial x}dx$
$u + \frac{\partial u}{\partial x}dx$

For large deflections, the effect of deflection on the strains has to be included in the strain-displacement relations. The accompanying figure shows a typical elongation of a linear element of length dx due to displacement in the x and z directions from bending.

Elongation = $dl \simeq \frac{\partial u}{\partial x}dx + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 dx$

The strain in the x direction is therefore

$$\varepsilon_x \simeq \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \qquad \text{(a)}$$

The strain in the y direction is likewise

$$\varepsilon_y \simeq \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \qquad \text{(b)}$$

The shearing strain due to displacements u and v is

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

It can be shown that the shearing strain due to displacement w is

$$\frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y}$$

The total shear strain in the middle plane is

$$\gamma_{xy} \simeq \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y} \qquad \text{(c)}$$

For small angles the displacements u and v can be represented by the change in displacement w as

$$u \simeq -z\frac{\partial w}{\partial x}$$

$$v \simeq -z\frac{\partial w}{\partial y}$$

Substitution into (a), (b) and (c) gives

$$\varepsilon_x \simeq -z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \qquad (2)^*$$

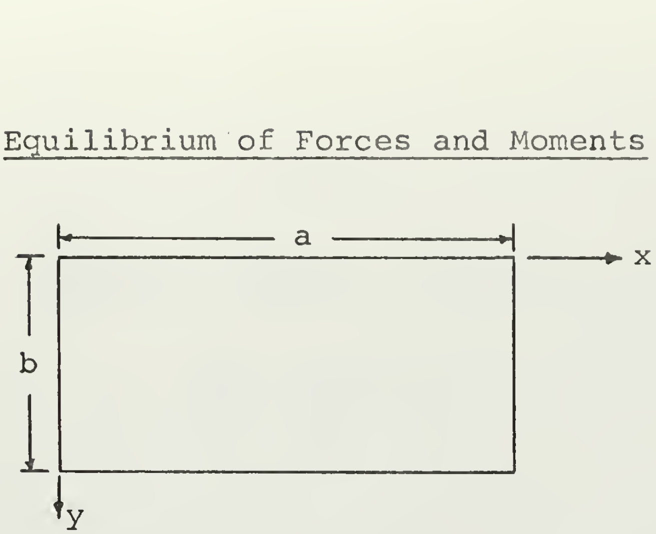
$$\varepsilon_y \simeq -z\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2$$

$$\gamma_{xy} \simeq -2z\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y}$$

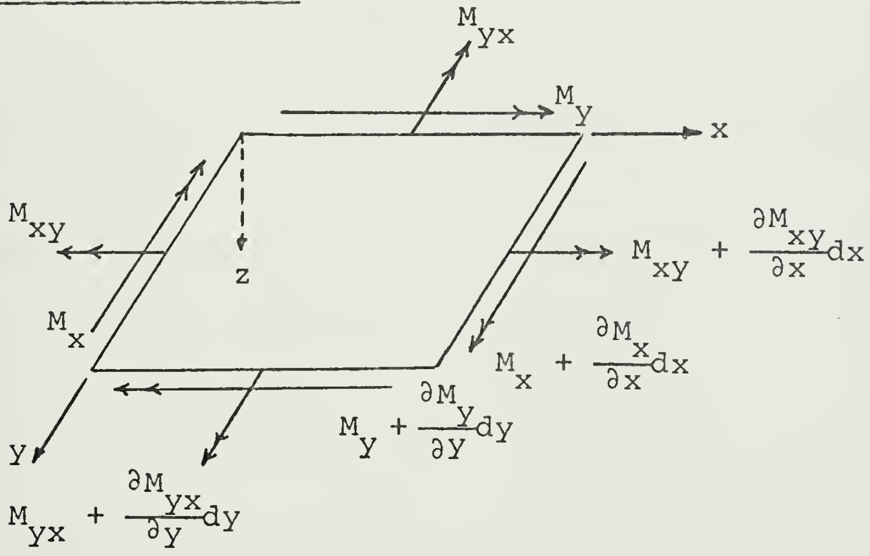
---

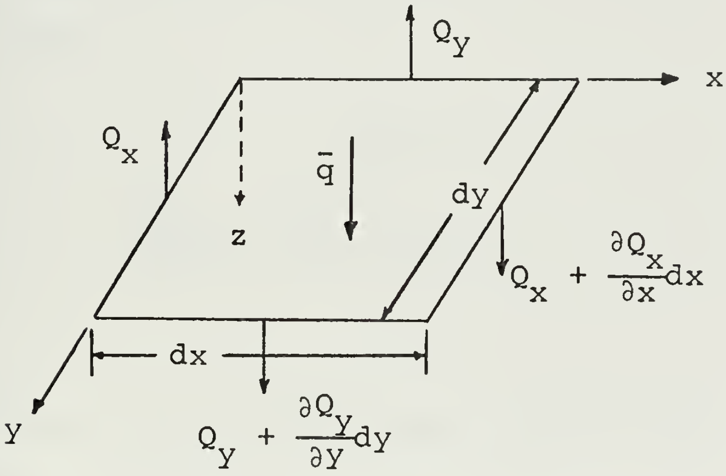
*Numbered equations correspond to those used in the main text.

Equilibrium of Forces and Moments



1. Lateral load $\bar{q}$ only


M_{yx}
M_y
x
M_{xy}
z
$M_{xy} + \frac{\partial M_{xy}}{\partial x}dx$
M_x
$M_x + \frac{\partial M_x}{\partial x}dx$
$M_y + \frac{\partial M_y}{\partial y}dy$
y
$M_{yx} + \frac{\partial M_{yx}}{\partial y}dy$


Q_y
x
Q_x
$\bar{q}$
z
dy
$Q_x + \frac{\partial Q_x}{\partial x}dx$
dx
y
$Q_y + \frac{\partial Q_y}{\partial y}dy$

The above diagrams are superimposed to make one loading condition. For equilibrium in the z-direction:

$$\Sigma F_z = 0$$

$$\frac{\partial Q_x}{\partial x}dxdy + \frac{\partial Q_y}{\partial y}dydx + \bar{q}\ dxdy = 0$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = -\bar{q} \qquad (a)$$

Moment equilibrium about the x-axis

$$\Sigma M_{xx} = 0$$

$$\frac{\partial M_{xy}}{\partial x}dxdy - \frac{\partial M_y}{\partial y}dydx + Q_y dxdy + \frac{\partial Q_y}{\partial y}dydxdy + \bar{q}dxdy0(dy) = 0$$

Neglecting the last two terms since they are of higher order gives

$$\frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} + Q_y = 0 \qquad (b)$$

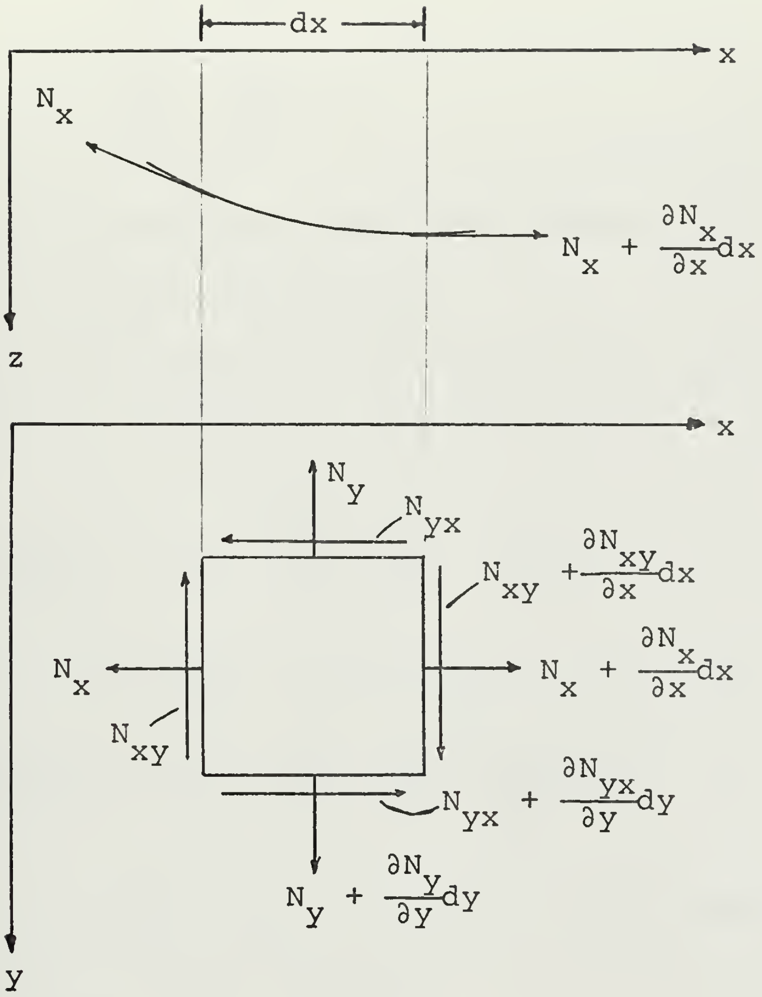
Moment equilibrium about the y-axis and similarly neglecting small terms

$$\Sigma M_{yy} = 0$$

$$\frac{\partial M_{yx}}{\partial y} + \frac{\partial M_x}{\partial x} - Q_x = 0$$

2. Forces in the middle plane of the plate (Membrane Stresses)

The forces acting in the middle of the plate may have a considerable effect on the bending of the plate and must be considered.


dx
x
z
N_x
$N_x + \frac{\partial N_x}{\partial x}dx$
x
y
N_y
N_{yx}
$N_{xy} + \frac{\partial N_{xy}}{\partial x}dx$
N_x
$N_x + \frac{\partial N_x}{\partial x}dx$
N_{xy}
$N_{yx} + \frac{\partial N_{yx}}{\partial y}dy$
$N_y + \frac{\partial N_y}{\partial y}dy$

Assuming no body forces or tangential forces

$$\Sigma F_x = 0$$

$$\frac{\partial N_x}{\partial x}dxdy + \frac{\partial N_{yx}}{\partial y}dydx = 0$$

$$\Sigma F_y = 0$$

$$\frac{\partial N_y}{\partial y}dydx + \frac{\partial N_{xy}}{\partial x}dxdy = 0$$

and from symmetry noting $N_{xy} = N_{yx}$

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \qquad (3)$$

Equations (3) are independent of equations (a), (b) and (c) and can be treated separately.

Projecting normal forces N_x onto the z axis and taking into account the bending of the plate and resulting small angles

$$-N_x dy\frac{\partial w}{\partial x} + (N_x + \frac{\partial N_x}{\partial x}dx)(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2}dx)dy$$

Similarly projecting normal forces N_y onto the z axis

$$-N_y dx\frac{\partial w}{\partial y} + (N_y + \frac{\partial N_y}{\partial y}dy)(\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2}dy)dx$$

Neglecting higher than second order terms gives

$$N_x\frac{\partial^2 w}{\partial x^2}dxdy + \frac{\partial N_x}{\partial x} \cdot \frac{\partial w}{\partial x}dxdy \qquad (d)$$

$$N_y\frac{\partial^2 w}{\partial y^2}dydx + \frac{\partial N_y}{\partial y} \cdot \frac{\partial w}{\partial y}dydx \qquad (e)$$

Treating the projections of the shearing forces onto the z axis the same way

(1) for N_{xy} with slope of deflection surface in the y direction on the two opposite sides of the element as $\frac{\partial w}{\partial y}$ and $\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial x \partial y}\,dx$

$$N_{xy}\frac{\partial^2 w}{\partial x \partial y}dxdy + \frac{\partial N_{xy}}{\partial x} \cdot \frac{\partial w}{\partial y}dxdy$$

(2) for N_{yx} with slope of deflection surface in the x direction on the two opposite sides of the element as $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x \partial y}dy$

$$N_{yx}\frac{\partial^2 w}{\partial x \partial y}dydx + \frac{\partial N_{yx}}{\partial y} \cdot \frac{\partial w}{\partial x}dydx$$

combining (1) and (2) gives

$$2N_{xy}\frac{\partial^2 w}{\partial x \partial y}dxdy + \frac{\partial N_{xy}}{\partial x}\frac{\partial w}{\partial y}dxdy + \frac{\partial N_{xy}}{\partial y}\frac{\partial w}{\partial x}dxdy \tag{f}$$

Differentiation of (b) with respect to y and of (c) with respect to x and substitution into (a) eliminates shearing forces Q_x and Q_y

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - \frac{\partial^2 M_{xy}}{\partial x \partial y} = -\bar{q}$$

Noting that $M_{yx} = -M_{xy}$, adding expressions (d), (e) and (f) to the load $\bar{q}dxdy$ originally defined and making use of (3)

$$\frac{\partial^2 M_x}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2}$$
$$= -(\bar{q} + N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y\frac{\partial^2 w}{\partial y^2}) \tag{4}$$

Moment - Curvature Relations

Substitution of equations (2) for ε_x, ε_y and γ_{xy} in equations (1) gives*

$$\sigma_x = \frac{E_x}{1-\nu_x\nu_y}\left[-z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\nu_y\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\nu_y\left(\frac{\partial w}{\partial y}\right)^2\right]$$

$$\sigma_y = \frac{E_y}{1-\nu_x\nu_y}\left[-z\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - z\nu_x\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\nu_x\left(\frac{\partial w}{\partial x}\right)^2\right] \qquad \text{(a)}$$

$$\tau_{xy} = -2zG\frac{\partial^2 w}{\partial x\partial y} + G\frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}$$

Taking the moments as defined before and integrating

$$M_x = \int_{-h/2}^{h/2} z\sigma_x dz$$

$$M_y = \int_{-h/2}^{h/2} z\sigma_y dz$$

$$M_{xy} = -\int_{-h/2}^{h/2} z\tau_{xy} dz$$

Even functions will drop out over the integration interval leaving the same expressions for moments M_x, M_y and M_{xy} as for a plate undergoing pure bending and only small deflections. The result is

---

*Numbers in parenthesis correspond to equations in the main text.

$$M_x = -D_x\left(\frac{\partial^2 w}{\partial x^2} + \nu_y\frac{\partial^2 w}{\partial y^2}\right)$$

$$M_y = -D_y\left(\frac{\partial^2 w}{\partial y^2} + \nu_x\frac{\partial^2 w}{\partial x^2}\right) \qquad (6)$$

$$M_{xy} = 2C\frac{\partial^2 w}{\partial x \partial y}$$

where the rigicity coefficients D_x and D_y are defined as

$$D_x \equiv \frac{E_x h^3}{12(1-\nu_x\nu_y)}$$

$$D_y \equiv \frac{E_y h^3}{12(1-\nu_x\nu_y)}$$

and

$$C \equiv \frac{Gh^3}{12}$$

"Equilibrium" Equation

Substitution of equations (6) into (4) gives

$$D_x\frac{\partial^4 w}{\partial x^4} + 2D_{xy}\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y\frac{\partial^4 w}{\partial y^4}$$

$$= \bar{q} + N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y\frac{\partial^2 w}{\partial y^2}$$

where

$$2D_{xy} = D_x\nu_y + D_y\nu_x + 4C$$

Substitution of the stress function defined by equations (5)

$$D_x\frac{\partial^4 w}{\partial x^4} + 2D_{xy}\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y\frac{\partial^4 w}{\partial y^4}$$

$$= \bar{q} + \frac{\partial^2 F}{\partial y^2}\cdot\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{\partial x \partial y}\cdot\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2} \qquad (7)$$

"Compatibility" Equation

The forces N_x, N_y, and N_{xy} in the middle plane of the plate depend on the strain due to bending as well as the external forces applied in the xy plane.

Again assuming no body forces and requiring load $\bar{q}$ is perpendicular, equations (3) apply for equilibrium in the middle xy plane

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \qquad (3)$$

The corresponding strain components are those of equations (2)

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$$

$$\varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \qquad (2)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}$$

Differentiating equations (2) and combining to eliminate u and v results in

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} \qquad (8)$$

Using Hooke's Law to relate strain and forces N

$$\varepsilon_x = \frac{N_x}{hE_x} - \nu_y \frac{N_y}{hE_y}$$

$$\varepsilon_y = \frac{N_y}{hE_y} - \nu_x \frac{N_x}{hE_x} \qquad (a)$$

$$\gamma_{xy} = \frac{N_{xy}}{hG}$$

Differentiation of equations (a) and substitution into equation (8) with use of equation (3)

$$J_x \frac{\partial^2 N_y}{\partial x^2} + 2J_{xy} \frac{\partial^2 N_{xy}}{\partial x \partial y} + J_y \frac{\partial^2 N_x}{\partial y^2}$$

$$= \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2}$$

where

$$J_x = \frac{1}{E_y h} \quad ; \quad J_y = \frac{1}{E_x h}$$

and

$$2J_{xy} = \frac{1}{Gh} - \nu_x J_y - \nu_y J_x$$

Substitution of the stress function defined by equations (5)

$$J_x \frac{\partial^4 F}{\partial x^4} + 2J_{xy} \frac{\partial^4 F}{\partial x^2 \partial y^2} + J_y \frac{\partial^4 F}{\partial y^4} = \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} \qquad (9)$$

APPENDIX B

EFFECTIVE WIDTH

Effective Width

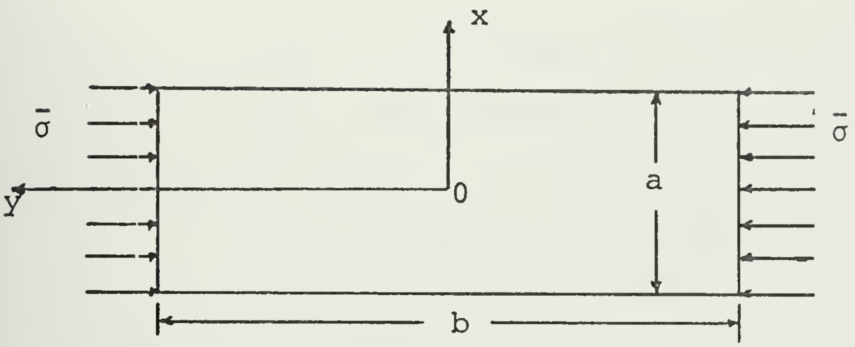


Figure a

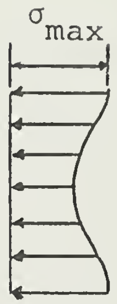


Figure b

Consider the simple case of a rectangular plate simply supported on all edges where the loaded edge remains straight in the plane of the plate at all times. If the load is below the buckling load, the stresses will be distributed evenly as shown in figure a. For post-buckling loads the center of the plate will exhibit less compressive strain than the edges because of large deflections of the center of the plate. The stress distribution is shown in figure b. The effective width relates the maximum stress σ_{max} uniformly distributed along a phantom plate sustaining the same total load as the real plate, thicknesses being the same. Therefore

$$a_e \sigma_{max} = a\bar{\sigma}$$

Where $\bar{\sigma}$ is the average edge stress. For this particular case

$$\frac{\bar{N}_y}{h} = \bar{\sigma} \quad \text{and} \quad \sigma_{max} = \sigma_{ye}$$

Where σ_{ye} is the edge membrane stress in the y-direction (σ_y at $x = \pm \frac{a}{2}$). Substitution into the above relation gives the result

$$a_e \sigma_{ye} = a \frac{\bar{N}_y}{h} \tag{34a}$$

Similarly, for loads in the x-direction

$$b_e \sigma_{xe} = b \frac{\bar{N}_x}{h} \tag{34b}$$

APPENDIX C

Details of Solution

1. Determination of Coefficients C_{pq}

Substitute deflection expression (37) into the right hand side of equation (16). The right hand side of equation (16) becomes*

Term #1

$$= \frac{h^2\pi^4}{a^2b^2}\{4\sin^2\frac{\pi}{a}x[b_1\sin\frac{2\pi}{b}y + 2b_2\sin\frac{4\pi}{b}y + 3b_3\sin\frac{6\pi}{b}y$$

$$+ 4b_4\sin\frac{8\pi}{b}y + 5b_5\sin\frac{10\pi}{b}y + 6b_6\sin\frac{12\pi}{b}y$$

$$+ 7b_7\sin\frac{14\pi}{b}y + 8b_8\sin\frac{16\pi}{b}y]^2\}$$

Term #2

$$- \frac{h^2\pi^4}{a^2b^2}\{4\cos^2\frac{\pi}{a}x[b_1 + b_1\cos\frac{2\pi}{b}y - b_2 + b_2\cos\frac{4\pi}{b}y + b_3$$

$$+ b_3\cos\frac{6\pi}{b}y - b_4 + b_4\cos\frac{8\pi}{b}y + b_5 + b_5\cos\frac{10\pi}{b}y - b_6$$

$$+ b_6\cos\frac{12\pi}{b}y + b_7 + b_7\cos\frac{14\pi}{b}y - b_8 + b_8\cos\frac{16\pi}{b}y]\cdot[b_1\cos\frac{2\pi}{b}y$$

$$+4b_2\cos\frac{4\pi}{b}y + 9b_3\cos\frac{6\pi}{b}y + 16b_4\cos\frac{8\pi}{b}y + 25b_5\cos\frac{10\pi}{b}y$$

$$+36b_6\cos\frac{12\pi}{b}y + 49b_7\cos\frac{14\pi}{b}y + 64b_8\cos\frac{16\pi}{b}y]\} \qquad \text{(a)}$$

---

*Note: since $m = 1$ for all cases, for convenience $b_{1,n}$ is represented by b_n.

Each of the two terms of equation (a) is multiplied out and like terms are collected in the form $\cos\frac{2q\pi}{b}y$ for $q = 0,1,2...16$ using the function product relations

$$4\sin\alpha\sin\beta = 2\cos(\alpha-\beta) - 2\cos(\alpha+\beta)$$

$$4\cos\alpha\cos\beta = 2\cos(\alpha-\beta) + 2\cos(\alpha+\beta)$$

The first and second terms produce coefficients $2X_q$ and $2Y_q$ respectively. Using the function relationships

$$\sin^2\frac{\pi}{a}x = \frac{1}{2} - \frac{1}{2}\cos\frac{2\pi}{a}x$$

$$\cos^2\frac{\pi}{a}x = \frac{1}{2} + \frac{1}{2}\cos\frac{2\pi}{a}x$$

there results

First term:

$$X_q\cos\frac{2q\pi}{b}y - X_q\cos\frac{2q\pi}{b}y\ \cos\frac{2\pi}{a}x$$

Second term:

$$Y_q\cos\frac{2q\pi}{b}y + Y_q\cos\frac{2q\pi}{b}y\ \cos\frac{2\pi}{a}x$$

Comparison with the right hand side of equation (17), the coefficients C_{pq} are

$$C_{o,q} = Y_q + X_q$$

$$C_{1,q} = Y_q - X_q$$

or

$$C_{pq} = Y_q + (-1)^p X_q$$

$$p = 0,1$$

$$q = 0,1,2\ldots16$$

2. Application of the Method of B. G. Galerkin to Equation (7)

Method of B.G. Galerkin (from reference [18])

It is required to determine the solution of the equation

$$L(w) = 0$$

where L is a differential operator in two variables, which solution satisfies homogeneous boundary conditions.

The approximate solution sought is in the form

$$\bar{w}(x,y) = \sum_{n=1}^{i} b_n F_n(x,y)$$

where $F_n(x,y)$ $(n = 1,2\ldots i)$ is a system of functions chosen to satisfy the boundary conditions and b_n are undetermined coefficients. Consider the functions $F_n(x,y)$ to be linearly independent over a given region (no one of the functions can be expressed as a linear combination of the others). For $\bar{w}(x,y)$ to be the exact solution of the given equation, it is necessary that

$$L(\bar{w}) = 0$$

If $L(\bar{w})$ is continuous, this requires that the expression $L(\bar{w})$ is orthogonal to all functions of the system $F_n(x,y)$ $(n = 1,2...i)$. With i constants, $b_1, b_2...b_i$, i conditions of orthogonality can be satisfied.

Requiring that the two functions are orthogonal over a region produces the following system of equations

$$\iint_R L(\bar{w})F_n(x,y)\,dxdy$$

$$= \iint_R L\left(\sum_n b_n F_n(x,y)\right)F_n(x,y)\,dxdy = 0$$

$$(n = 1,2...i)$$

from which the coefficients b_n can be determined. These coefficients will define the solution $\bar{w}(x,y)$.

The method of B.G. Galerkin can also be obtained from the principle of virtual work.

Application of the method of Galerkin to equation (7) requires that the following equation be satisfied by all functions $f_r(x)g_s(y)$

$$\int_0^{a/2}\int_0^{b/2}\Big[D_x\frac{\partial^4 w}{\partial x^4} + 2D_{xy}\frac{\partial^4 w}{\partial x^2\partial y^2} + D_y\frac{\partial^4 w}{\partial y^4}$$

$$- \bar{q} - \frac{\partial^2 F}{\partial y^2}\cdot\frac{\partial^2 w}{\partial x^2} + 2\,\frac{\partial^2 F}{\partial x\partial y}\,\frac{\partial^2 w}{\partial x\partial y} - \frac{\partial^2 F}{\partial x^2}\,\frac{\partial^2 w}{\partial y^2}\Big]$$

$$X\ f_r(x)g_s(y)\,dxdy = 0 \qquad (21)$$

Substitution of w and F, expressions (15) and (20) respectively

$$
\begin{aligned}
&\int_0^{a/2}\int_0^{b/2} \sum_r \sum_s \{hD_x \sum_m \sum_n \left(\frac{m\pi}{a}\right)^4 b_{mn}\left[(-1)^{n+1} + \cos\frac{2n\pi}{b}y\right]\cos\frac{m\pi}{a}x \\
&+ 2\, D_{xy} h \sum_{m}\sum_{n} \left(\frac{2n\pi}{b}\right)^2 \left(\frac{m\pi}{a}\right)^2 b_{mn} \cos\frac{2n\pi}{b}y \cos\frac{m\pi}{a}x \\
&+ h\, D_y \sum_m \sum_n \left(\frac{2n\pi}{b}\right)^4 b_{mn} \cos\frac{2n\pi}{b}y \cos\frac{m\pi}{a}x - \bar{q} \\
&- h^3 \sum_m \sum_n \sum_p \sum_q \left(\frac{m\pi}{a}\right)^2 \left(\frac{2q\pi}{b}\right)^2 \phi_{pq} b_{mn}\left[(-1)^{n+1} + \cos\frac{2n\pi}{b}y\right] \\
&\qquad \cdot \cos\frac{2q\pi}{b}y \cos\frac{m\pi}{a}x \cos\frac{2p\pi}{a}x \\
&- \bar{N}_y h \sum_m \sum_n \left(\frac{2n\pi}{b}\right)^2 b_{mn} \cos\frac{2n\pi}{b}y \cos\frac{m\pi}{a}x \\
&- h^3 \sum_m \sum_n \left(\frac{2n\pi}{b}\right)^2 \left(\frac{2p\pi}{a}\right)^2 \phi_{pq} b_{mn} \cos\frac{2n\pi}{b}y \cos\frac{2q\pi}{b}y \cos\frac{m\pi}{a}x \cos\frac{2p\pi}{a}x \\
&- \bar{N}_x h \sum_m \sum_n \left(\frac{m\pi}{a}\right)^2 b_{mn}\left[(-1)^{n+1} + \cos\frac{2n\pi}{b}y\right]\cos\frac{m\pi}{a}x \\
&- 2\bar{S} h \sum_m \sum_n \left(\frac{2n\pi}{b}\right)\left(\frac{m\pi}{a}\right) b_{mn} \sin\frac{2n\pi}{b}y \sin\frac{m\pi}{a}x \\
&+ 2h^3 \sum_m \sum_n \sum_p \sum_q \left(\frac{2n\pi}{b}\right)\left(\frac{m\pi}{a}\right)\left(\frac{2p\pi}{a}\right)\left(\frac{2q\pi}{b}\right) \phi_{pq} b_{mn} \sin\frac{2n\pi}{b}y \sin\frac{2q\pi}{b}y \\
&\cdot \sin\frac{m\pi}{a}x \sin\frac{2p\pi}{a}x\} \cdot \left[(-1)^{s+1} + \cos\frac{2s\pi}{b}y\right]\cos\frac{r\pi}{a}x \, dxdy = 0
\end{aligned}
$$

$$m,r = 1,3,5\ldots.$$

$$n,s = 1,2,3\ldots.$$

The orthogonality properties of the trigonometric functions must be used to evaluate the integral of equation (21). By employing the function relations

$$\cos A + \cos B = 2\cos\tfrac{1}{2}(A + B)\cos\tfrac{1}{2}(A - B)$$
$$\sin A + \sin B = 2\sin\tfrac{1}{2}(A + B)\cos\tfrac{1}{2}(A - B)$$

The trigonometric products may be expressed in typical forms

$$\int_0^{a/2} \cos\frac{m\pi}{a}x \, \cos\frac{r\pi}{a}x \, dx$$

$$= \frac{1}{4}\int_0^{a/2} [2\cos\frac{\pi}{a}(m+r)x + \cos\frac{\pi}{a}(m-r)x + \cos\frac{\pi}{a}(r-m)x]dx$$

and

$$\int_0^{a/2} \sin\frac{m\pi}{a}x \, \cos\frac{r\pi}{a}x \, dx$$

$$= \frac{1}{4}\int_0^{a/2} [2\sin\frac{\pi}{a}(m+r)x + \sin\frac{\pi}{a}(m-r)x - \sin\frac{\pi}{a}(r-m)x]dx$$

where integration is carried out for all m and r. By combining terms that integrate to similar values, these integrals are equivalent to

$$\frac{1}{4}\int_0^{a/2} [2\cos\frac{\pi}{a}|m+r|x + 2\cos\frac{\pi}{a}|m-r|x + 2\cos\frac{\pi}{a}|r-m|x]dx$$

and

$$\frac{1}{4}\int_0^{a/2} [2\sin\frac{\pi}{a}|m+r|x + 2\sin\frac{\pi}{a}|m-r|x - 2\sin\frac{\pi}{a}|r-m|x]dx$$

where the second two terms in each expression integrate to

zero if $r > m$ or $m > r$ respectively. Manipulation of the trigonometric products on the right hand side of equation (21) into these typical relations permits the reduction of the equation into the convenient form of equation (22). Table 3 lists all the specific integrals from equation (21) with their respective values.

APPENDIX D

COMPUTER PROGRAMS

The computer program to solve the simultaneous equations (39) is written in Fortran IV Level G1 for the IBM system 370. The program consists of the main and three subprograms--MATHLIB subroutine ZEROIN, and subroutines EVAL and CPQ. Table 4 lists the important computer symbols used.

ZEROIN is a Fortran IV subprogram, from the M.I.T. mathematical library, which computes the solution vector $\underline{b}$ of a set of N simultaneous, nonlinear equations $\sum_{1}^{N} \underline{F}_n(\underline{b}) = 0$ using double-precision arithmetic. $\underline{b}$ is obtained by an iterative method beginning with estimated values of the solution vector and iteration is performed as

$$\underline{b}^{k+1} = \underline{b}^k + \underline{D}^k$$

with the vector $\underline{D}^k = J^k \cdot \underline{F}(\underline{b}^k)$ where $\underline{F}(\underline{b}^k)$ represents the vector of function values at the point $\underline{b}^k$ and J^k represents an approximation to the Jacobian at $\underline{b}^k$. Convergence is tested with the expression

$$|\underline{b}^{k+1} - \underline{b}^k|^2 \leq 0.5 \times 10^{-12} |\underline{b}^{k+1}|^2$$

Subroutine EVAL generates the eight simultaneous equations and calculates the vector of function values for a given solution vector $\underline{b}$. EVAL is called repeatedly by subroutine ZEROIN.

Subroutine CPQ is in turn called by subroutine EVAL, with a solution vector b, to calculate the coefficients C_{pq} that are used in the simultaneous equations.

For the starting values of the solution vector b, the main program uses a well guessed set of values set in the program. If the lateral load is not zero ($Q^* \neq 0$) (see note 1) or the present loading conditions did not change significantly from the last set, the previously computed solution vector b is used for the starting values.

If ZEROIN cannot achieve convergence in 50 iterations, the main program alters the initial set of starting values and returns them to ZEROIN. Normally convergence is obtained within the first 50 iterations, however, never more that a second set of values is required.

Part 1 - Design Charts

The main program varies the nondimensional loads Q* and N* or S* for a particular set of plate parameters. Tables of nondimensional deflection coefficients, nondimensional deflection at the center of the plate, nondimensional effective width and nondimensional bending moment (y-direction) at the middle of the supports ($y = \pm b/2$) are outputs. Additionally, CALCOMP plots of the last three tables are produced.

Part 2 - Behaviour Programs

The main program reads the input parameters for 4 sets of loading conditions on a particular plate and outputs CALCOMP plots of nondimensional plate deflection along the centerline ($x = 0$) and nondimensional total principal stresses along the centerlines ($x = 0$ and $y = 0$). These programs are completely general and can be used with any set of loading conditions.

The design programs must be used with care. With reference to equations (36a) and (36b), the effective width is only defined for $N_y^* \neq 0$ or $N_x^* \neq 0$ respectively. Hence, variation of S* with $N^* = 0$ will give a value of zero for the effective width. The design programs are written so that no computation or plot of effective width will be made for any variation of S* with N* fixed. Additionally, deflection at the center of the plate and the y-direction bending moment may not be significant parameters in this case. It is recommended that the "behaviour" program be used if variation of the shear load, S*, is desired.

Program Descriptions

Main Program 1 - Design ($N_y^* = 0$)

Input one card per set of complete design curves set up as follows:

Column	Parameter
1-10	RHO
11-20	GAMMA
21-30	ETA
31-40	*NXSTAR (Enter Negative Value to vary)
41-50	*SSTAR (Enter negative value to vary)
51-60	RANG
61-70	RANGQ

Note: Either N_x^* or S may be varied, but not both in the same run.

Output

1. Tables
   - a. $b_{1,n}$
   - b. $\frac{w}{h}$ at $y = x = 0$
   - c. b_e (only if N_x^* is varied)
   - d. M_y^* at $y = \pm \frac{b}{2}, x = 0$

2. CALCOMP Plots

   Tables b, c and d above.

3. Values

   N_x^* or S^* = 0 to RANG in 10 increments of RANG/20 and

   Q^* = 0, RANGQ/6, RANGQ/3, RANGQ

Termination

A blank card as the last data card is required.

Main Program 2 - Design ($N_x^* = 0$)

Input one card per set of complete design curves set up as follows:

Column	Parameter
1-10	RHO
11-20	GAMMA
21-30	ETA
31-40	*NYSTAR (Enter negative value to vary)
41-50	*SSTAR (Enter negative value to vary)
51-60	RANG
61-70	RANGQ

*Note: Either N_y^* or S^* may be varied, but not both in the same run.

Output

1. Tables
    a. $b_{1,n}$
2. CALCOMP Plots
    a. $\frac{w}{h}$ at $y = x = 0$
    b. a_e (only if N_y^* is varied)
    c. M_y^* at $y = \pm \frac{b}{2}$, $x = 0$

3. Values

N_y^* or S* = 0 to RANG in 10 increments of RANG/20 and

Q* = 0, RANGQ/6, RANGQ/3, RANGQ

Termination

A blank card as the last data card is required.

Main Program 3 - Behaviour

Input four cards per program run, each set up as follows:

Column	Parameter	
1-10	RHO	must be same all 4 cards
11-20	GAMMA	
21-30	ETA	
31-40	NSTAR (see note 2)	
41-50	SSTAR	
51-60	QSTAR	
61-70	*TB	

*Note: If TB other than zero, stresses at top of plate will be computed. TB = zero defaults to bottom of plate.

Output

1. Tables
    a. $b_{1,n}$
    b. $\frac{w}{h}$ at $y = x = 0$
    c. σ_1^* and σ_2^* at $y = x = 0$
2. CALCOMP Plots
    a. $\frac{w}{h}$ at $x = 0$ for $-\frac{1}{2} \leq \frac{y}{b} \leq +\frac{1}{2}$
    b. σ_1^* and σ_2^* at $x = 0$ for $-\frac{1}{2} \leq \frac{y}{a} \leq +\frac{1}{2}$

c. σ_1^* and σ_2^* at $y = 0$ for $-\frac{1}{2} \leq \frac{x}{a} \leq +\frac{1}{2}$

Termination

A blank card as the fifth card is required.

Subroutine EVAL

Two subroutine programs EVAL - one for $N_y^* = 0$, the other for $N_x^* = 0$. The appropriate EVAL program must be used with the corresponding main programs.

Subroutine ZEROIN & CPQ

Completely general subprograms. Use with all combinations of main programs and EVAL.

NOTE 1

If the lateral load is equal to zero ($Q^* = 0$) a possible but trivial solution is that all the coefficients $b_{mn} = 0$. Care must be exercised to avoid using $b_{mn} = 0$ for starting values in this case. Additionally, under certain circumstances the solution readily converges to zero rather than the appropriate non-trivial solution. The programmer should be aware of this possibility.

NOTE 2

The behaviour program may be used with either N_y^* or N_x^* as a loading, but not both. The main program must be used with the appropriate subroutine EVAL.

SAMPLE INPUT

DESIGN PROGRAM

```
0.6666667  1.0       1.0         -1.0        0.0         12.0        8.0
(Blank card)
```

BEHAVIOUR PROGRAM

```
0.6666667  1.0       1.0         0.0         13.0        0.0
0.6666667  1.0       1.0         0.0         14.0        0.0
0.6666667  1.0       1.0         0.0         16.0        0.0
0.6666667  1.0       1.0         0.0         20.0        0.0
(Blank card)
```

MAIN PROGRAM 1 - DESIGN

$(N_Y^* = 0)$

```
C   CALCULATION OF NONDIMENSIONAL DEFLECTION COEFFICIENTS OF A LARGE ASPECT
C   RATIO ORTHOTROPIC PLATE UNDER COMBINED LOADINGS
C   ***  USE THIS PROGRAM TO VARY INPLANE LOAD NXSTAR  ***
C   B(N)     =  NONDIMENSIONAL DEFLECTION COEFFICIENTS
C   RHO      =  VIRTUAL ASPECT RATIO OF ORTHOTROPIC PLATE
C   ETA      =  TORSION COEFFICIENT OF ORTHOTROPIC PLATE
C   GAMMA    =  ORTHOTROPIC PLATE COEFFICIENT
C   QSTAR    =  NONDIMENSIONAL HYDROSTATIC PRESSURE LOADING
C   NSTAR    =  NONDIMENSIONAL AVERAGE INPLANE COMPRESSIVE LOADING (X-DIRECTION)
C   SSTAR    =  NONDIMENSIONAL INPLANE EDGE SHEAR LOADING (ALL EDGES)
C   CPQ      =  COEFFICIENT, QUADRATIC FUNCTIONS OF B(N)
C   APQ      =  COEFFICIENT
C   W/H      =  NONDIMENSIONAL DEFLECTION OF CENTER OF PLATE
C   BE/B     =  NONDIMENSIONAL EFFECTIVE WIDTH
C   MYSTAR   =  NONDIMENSIONAL BENDING MOMENT AT MIDDLE OF SUPPORTS
C   ITERATIVE SOLUTION OF EIGHT SIMULTANEOUS NONLINEAR CUBIC EQUATIONS
C   ( F(B)=0 ), USING THE MATHLIB SUBPROGRAM ZEROIN (AP-21).
C   SUBPROGRAM EVAL CALCULATES F(B) FOR A VECTOR OF B(N) VALUES.
C   SUBPROGRAM CPQ CALCULATES COEFFICIENTS CPQ OF USE IN THE
C   SIMULTANEOUS EQUATIONS.
C   OUTPUT ARE GRAPHS OF W/H, BE/B AND MYSTAR FOR QSTAR = 0,1,2,3 TIMES RANGE/3
      REAL*8 RHO,GAMMA,ETA,NSTAR,QSTAR,SSTAR
      REAL*8 R,EPS,D,C,VARY,RANG,DET,RANGQ,DETQ
      DOUBLE PRECISION B(8),F(8)
      DIMENSION XAX(2),YAX1(1),YAX2(1),YAX3(2),XAX1(1),XAX2(1)
      REAL QS(4),NS(11),BB(8,44),W(11,4),BE(10,4),MYSTAR(11,4),PS(10)
      REAL WO1(11),WO2(11),WO3(11),WO4(11),BE1(10),BE2(10),BE3(10)
      REAL BE4(10),MY1(11),MY2(11),MY3(11),MY4(11),NST
      COMMON RHO,GAMMA,ETA,NSTAR,QSTAR,SSTAR
      EXTERNAL EVAL
      DATA KI,KO/5,6/
      DATA XAX1/' NST'/,XAX2/' SST'/,XAX(2)/'AR  '/
      DATA YAX1/'W/H '/,YAX2/'BE/B'/,YAX3/' MYS','TAR '/
      DATA K0,K2,K4,K6/240,241,242,243/
      DATA SI/0.07/,PI/3.141593/
      AEE=3.0*(1.0-0.09)
```

```
      YS=4.0*PI**2
C   SUPPRESS ERRORS CAUSED BY ZEROIN SUBROUTINE IN PROCESS OF CONVERGING
C   TO A SOLUTION.
      CALL ERRSET (208,256,-1,1,1,209)
C   USE CALCOMP PLOTTER SUBROUTINES
      CALL NEWPLT('M10347','9782','VELLUM ','BLACK')
   50 CONTINUE
      MOUSE=0
      NN=1
C   ENTER VIRTUAL ASPECT RATIO, ORTHOTROPIC MATERIAL CONSTANTS AND
C   LOADINGS.  ENTER NEGATIVE VALUE IN PLACE OF PARTICULAR LOAD TO
C   BE VARIED.  FIX THE OTHER LOAD AS A CONSTANT.  ENTER MAX VALUE OF
C   LOAD TO BE VARIED (DEFAULT IS 20.0) AND MAX VALUE OF QSTAR (DEFAULT
C   IS 6.0).
      READ(KI,100)RHO,GAMMA,ETA,NSTAR,SSTAR,RANG,RANGQ
      IF(RHO.EQ.0.0) GO TO 51
      NST=NSTAR
      SST=SSTAR
      IF(NST.LT.0.0) XAX(1)=XAX1(1)
      IF(SST.LT.0.0) XAX(1)=XAX2(1)
      IF(RANG.NE.0.0) DET=RANG/2.D1
      IF(RANG.EQ.0.0) DET=1.D0
      IF(RANGQ.NE.0.0) DETQ=RANGQ/6.D0
      IF(RANGQ.EQ.0.0) DETQ=1.D0
C   INITIALIZE NONDIMENSIONAL DEFLECTION COEFFICIENTS B(N)
      B(1)=0.4D0
      DO 12 I=1,7
      B(I+1)=(-1.D0)**I*B(I)*0.5D0
   12 CONTINUE
C   STORE SET OF B(N) VALUES AS F(N)
      DO 13 I=1,8
      F(I)=B(I)
   13 CONTINUE
      WRITE(KO,250)
C   INITIALIZE LOADING CONSTANTS
      QSTAR=0.D0
```

```
C   VARY QSTAR
      DO 90 IQ=1,4
      VARY=0.D0
      IF(NST.LT.0.0)NSTAR=VARY
      IF(SST.LT.0.0)SSTAR=VARY
C   VARY NSTAR OR SSTAR
      DO 91 II=1,11
      IF(IQ.NF.1.AND.II.NE.1) GO TO 19
C   RESET INITIAL VALUES P(N)
      DO 17 I=1,8
      B(I)=F(I)
   17 CONTINUE
   19 CONTINUE
      IN=0
      D=1.1
      C=0.2D0
      GO TO 14
   15 CONTINUE
      IN=IN+1
      IF(IN.GE.20) GO TO 16
      D=C+D
      DO 14 I=1,8
      B(I)=F(I)/D
   14 CONTINUE
      IRITE=2
      R=0.4D0
      EPS=0.5D-12
C   USE MATHLIB SUBROUTINE TO SOLVE THE NONLINEAR EQUATIONS
      CALL ZEROIN(8,B,R,EPS,ICV,EVAL,IRITE)
      IF(ICV.GE.2) GO TO 15
      WRITE(KO,240)QSTAR,NSTAR,SSTAR,B,IN
      GO TO 92
   16 CONTINUE
      WRITE(KO,206)NN
      MOUSE=1
      GO TO 94
```

```
   92 CONTINUE
      SUM=0.0
      V=B(1)
C     STORE COMPUTED VALUES OF B(N) AS BB(N,NN)
      DO 18 N=1,8
C     CHANGE SIGN OF DEFLECTION IF OTHER BUCKLING POSITION WAS COMPUTED
      IF(QSTAR.FQ.0.0.ANC.SSTAR.EQ.0.0.AND.V.LT.0.0) B(N)=-B(N)
      BB(N,NN)=B(N)
C     COMPUTE BENDING MOMENT AT MIDDLE OF SUPPORTS
      YSTAR=YS*B(N)*N**2*(-1)**N
      SUM=SUM+YSTAR
   18 CONTINUF
      NN=NN+1
      MYSTAR(II,IQ)=SUM
C     COMPUTE DEFLECTION AT CENTER OF PLATE
      W(II,IQ)=2.0*(B(1)+B(3)+B(5)+B(7))
C     COMPUTE EFFECTIVE WIDTH
      IF(NSTAR.EQ.0.0) GC TO 93
      SUM=0.0
      DO 85 IIQ=1,16
      CALL CPQ(IIQ,B,C1,C0)
      BEE=(-1.0)**IIQ*CO/IIQ**2
      SUM=SUM+BEE
   85 CONTINUE
      BE(II-1,IQ)=NSTAR/(NSTAR+AEE*SUM)
      WRITE(KO,230)W(II,IQ),MYSTAR(II,IQ),BE(II-1,IQ)
      GO TO 94
   93 CONTINUE
      WRITE(KO,230)W(II,IQ),MYSTAR(II,IQ)
   94 CONTINUF
      NS(II)=VARY*DET
      VARY=VARY+2.D0
      IF(NST.LT.0.0)NSTAR=VARY*DET
      IF(SST.LT.0.0)SSTAR=VARY*DET
   91 CONTINUE
      QS(IQ)=QSTAR
```

```
      QSTAR=QSTAR+2.D0*DETQ
   90 CONTINUE
C  NO PLOT IF A SOLUTION DID NOT CONVERGE
      IF(MOUSE.EQ.1) GO TO 50
C  OUTPUT DATA
      WRITE(KO,200)
      WRITE(KO,201)RHO,GAMMA,ETA
      IF(NST.LT.0.0)WRITE(KO,202)SSTAR
      IF(SST.LT.0.0)WRITE(KO,222)NSTAR
      IF(NST.LT.0.0)WRITE(KO,203)
      IF(SST.LT.0.0)WRITE(KO,233)
      NN=1
      DO 30  IQ=1,4
      WRITE(KO,204)QS(IQ)
      DO 31  II=1,11
      WRITE(KO,205)NS(II),(BB(N,NN),N=1,8)
      NN=NN+1
   31 CONTINUE
   30 CONTINUE
      WRITE(KO,300)RHO,GAMMA,ETA,SSTAR
      WRITE(KO,301)(QS(IQ),IQ=1,4)
      WRITE(KO,302)
      WRITE(KO,303)((NS(II),(W(II,IQ),IQ=1,4)),II=1,11)
      WRITE(KO,304)
      WRITE(KO,305)((NS(II),(MYSTAR(II,IQ),IQ=1,4)),II=1,11)
      WRITE(KO,306)
      WRITE(KO,307)((NS(II+1),(BE(II,IQ),IQ=1,4)),II=1,10)
C  NUMBER OF VALUES OF EFFECTIVE WIDTH TO BE PLOTTED
      MM=1
      DO 89 I=1,9
      IF(BE(I,1).EQ.1.0.AND.BE(I+1,1).NE.1.0) MM=I
      IF(MM.GT.1) GO TO 20
   89 CONTINUE
   20 CONTINUE
      NO=11-MM
C  ARRANGE DATA TO BE PLOTTED INTO VECTORS
```

```
      DO 40 I=1,11
      WO1(I)=W(I,1)
      WO2(I)=W(I,2)
      WO3(I)=W(I,3)
      WO4(I)=W(I,4)
      MY1(I)=MYSTAR(I,1)
      MY2(I)=MYSTAR(I,2)
      MY3(I)=MYSTAR(I,3)
      MY4(I)=MYSTAR(I,4)
   40 CONTINUE
      IF(SST.LT.0.0) GO TO 41
      DO 41 I=1,NO
      PS(I)=NS(I+MM)
      BE1(I)=BE(I+MM-1,1)
      BE2(I)=BE(I+MM-1,2)
      BE3(I)=BE(I+MM-1,3)
      BE4(I)=BE(I+MM-1,4)
   41 CONTINUE
C     PLCT NONDIMENSIONAL PLATE DEFLECTIONS
      CALL PICTUR(4.,4.,XAX,8,YAX1,4,NS,WO1,-11,SI,KO,NS,WO2,-11,SI,K2,
     1 NS,WO3,-11,SI,K4,NS,WO4,-11,SI,K6)
      IF(SST.LT.0.0) GO TO 42
C     PLOT NONDIMENSIONAL EFFECTIVE WIDTH
      CALL PICTUR(4.,4.,XAX,8,YAX2,4,PS,BE1,-NO,SI,KO,PS,BE2,-NO,SI,K2,
     1 PS,BE3,-NO,SI,K4,PS,BE4,-NO,SI,K6)
   42 CONTINUF
C     PLOT NONDIMENSIONAL BENDING MCMFNT IN THE Y-DIRECTION
      CALL PICTUR(4.,4.,XAX,8,YAX3,8,NS,MY1,-11,SI,KO,NS,MY2,-11,SI,K2,
     1 NS,MY3,-11,SI,K4,NS,MY4,-11,SI,K6)
      WRITE(KO,299)
      GO TO 50
   51 CONTINUF
      CALL ENDPLT
      WRITF(KO,207)
  100 FORMAT(8F10.3)
  200 FORMAT('1'//,28X,'NONDIMENSIONAL DEFLECTION COFFFICIENTS OF AN',
```

```
   1 ' ORTHOTROPIC PLATE'//)
201 FORMAT(38X,'RHO =',F6.3,2X,' GAMMA =',F6.3,2X,' ETA =',F6.3,2X/)
202 FORMAT(52X,'SSTAR =',F6.2/)
222 FORMAT(52X,'NSTAR =',F6.2/)
203 FORMAT(5X,'NXSTAR',9X,'B11',10X,'B12',10X,'B13',10X,'B14',
   1 10X,'B15',10X,'B16',10X,'B17',10X,'B18'/)
233 FORMAT(5X,'SSTAR',10X,'B11',10X,'B12',10X,'B13',10X,'B14',
   1 10X,'B15',10X,'B16',10X,'B17',10X,'B18'/)
204 FORMAT(3X,'QSTAR =',F5.1)
205 FORMAT(4X,F6.2,5X,8(F10.6,3X))
206 FORMAT(5X,'SOLUTION DID NOT CONVERGE AT STAGE',I3/10X,'NO PLOT')
207 FORMAT('1','TERMINATION OF RUN')
230 FORMAT(3F10.5)
240 FORMAT(11F10.5,I4)
250 FORMAT('1',2X,'QSTAR',5X,'NSTAR',5X,'SSTAR',7X,'B11',7X,'B12',7X,
   1 'B13',7X,'B14',7X,'B15',7X,'B16',7X,'B17',7X,'B18'/4X,'W/H',
   2 6X,'MYSTAR',5X,'BE/B'/)
299 FORMAT('1',20X,'END DATA SET')
300 FORMAT('1'///9X,'RHO =',F6.3,2X,' GAMMA =',F6.3,2X,' ETA =',
   1 F6.3//23X,'SSTAR =',F6.2//)
301 FORMAT(36X,'QSTAR'//9X,'NXSTAR',5X,4(F6.2,4X)///)
302 FORMAT(22X,4('W/H',7X)/)
303 FORMAT(8X,F6.2,6X,F7.3,3X,F7.3,3X,F7.3,3X,F7.3)
304 FORMAT(/21X,4('MYSTAR',4X)/)
305 FORMAT(8X,F6.2,6X,F8.3,2X,F8.3,2X,F8.3,2X,F8.3)
306 FORMAT(/21X,4('BE/B',6X)/)
307 FORMAT(8X,F6.2,6X,F6.3,4X,F6.3,4X,F6.3,4X,F6.3)
    STOP
    END
```

MAIN PROGRAM 2 - DESIGN

$(N_x^* = 0)$

[illegible]

[illegible] - [illegible]

([illegible])

```
C  CALCULATION OF NONDIMENSIONAL DEFLECTION COEFFICIENTS OF A LARGE ASPECT
C  RATIO ORTHOTROPIC PLATE UNDER COMBINED LOADINGS
C  B(N)    =  NONDIMENSIONAL DEFLECTION COEFFICIENTS
C  RHO     =  VIRTUAL ASPECT RATIO OF ORTHOTROPIC PLATE
C  ETA     =  TORSION COEFFICIENT OF ORTHOTROPIC PLATE
C  GAMMA   =  ORTHOTROPIC PLATE COEFFICIENT
C  QSTAR   =  NONDIMENSICNAL HYDROSTATIC PRESSURE LOADING
C  NSTAP   =  NONDIMENSICNAL AVERAGE INPLANE COMPRESSIVE LOADING (Y-DIRECTION)
C  SSTAR   =  NONDIMENSICNAL INPLANE EDGE SHEAR LOADING (ALL EDGES)
C  CPQ     =  COEFFICIENT, QUADRATIC FUNCTIONS OF B(N)
C  APQ     =  COEFFICIENT
C  W/H     =  NONDIMENSIONAL DEFLECTION OF CENTER OF PLATE
C  AE/A    =  NONDIMENSICNAL EFFECTIVE WIDTH
C  MYSTAR  =  NONDIMENSIONAL BENDING MOMENT AT MIDDLE OF SUPPORTS
C  ITERATIVE SOLUTION OF EIGHT SIMULTANEOUS NONLINEAR CUBIC EQUATIONS
C  ( F(B)=0 ), USING THE MATHLIB SUBPROGRAM ZEROIN (AP-21).
C  SUBPROGRAM EVAL CALCULATES F(B) FOR A VECTOR OF B(N) VALUES.
C  SUBPROGRAM CPO CALCULATES COEFFICIENTS CPQ OF USE IN THE
C  SIMULTANEOUS EQUATIONS.
C  OUTPUT ARE GRAPHS OF W/H, AE/A AND MYSTAR FOR QSTAR = 0,1,2,3 TIMES RANGE/3
      REAL*8 RHO,GAMMA,ETA,NSTAR,QSTAR,SSTAR
      REAL*8 R,EPS,D,C,VARY,RANG,DET,RANGQ,DETQ
      DOUBLE PRECISION B(8),F(8)
      DIMENSION XAX(2),YAX1(1),YAX2(1),YAX3(2),XAX1(1),XAX2(1)
      REAL QS(4),NS(11),BB(8,44),W(11,4),AE(10,4),MYSTAR(11,4),PS(10)
      REAL WO1(11),WO2(11),WO3(11),WO4(11),AE1(10),AE2(10),AE3(10)
      REAL AE4(10),MY1(11),MY2(11),MY3(11),MY4(11),NST
      COMMON RHO,GAMMA,ETA,NSTAR,QSTAR,SSTAR
      EXTERNAL EVAL
      DATA KI,KO/5,6/
      DATA XAX1/' NST'/,XAX2/' SST'/,XAX(2)/'AR  '/
      DATA YAX1/'W/H '/,YAX2/'AE/A'/,YAX3/' MYS','TAR '/
      DATA KO,K2,K4,K6/240,241,242,243/
      DATA SI/0.07/,PI/3.141593/
      AEE=3.0*(1.0-0.09)
      YS=4.0*PI**2
```

```
C  SUPPRESS ERRORS CAUSED BY ZEROIN SUBROUTINE IN PROCESS OF CONVERGING
C  TO A SOLUTION.
      CALL ERRSET (208,256,-1,1,1,209)
C  USE CALCOMP PLOTTER SUBROUTINES
      CALL NEWPLT('M10347','9782','VELLUM ','BLACK')
   50 CONTINUE
      MOUSE=0
      NN=1
C  ENTER VIRTUAL ASPECT RATIO, ORTHOTROPIC MATERIAL CONSTANTS AND
C  LOADINGS.  ENTER NEGATIVE VALUE IN PLACE OF PARTICULAR LOAD TO
C  BE VARIED.  FIX THE OTHER LOAD AS A CONSTANT.  ENTER MAX VALUE OF
C  LOAD TO BE VARIED (DEFAULT IS 20.0) AND MAX VALUE OF QSTAR (DEFAULT
C  IS 6.0).
      READ(KI,10)RHO,GAMMA,ETA,NSTAR,SSTAR,RANG,RANGQ
      IF(RHO.EQ.0.0) GO TO 51
      NST=NSTAR
      SST=SSTAR
      IF(NST.LT.0.0) XAX(1)=XAX1(1)
      IF(SST.LT.0.0) XAX(1)=XAX2(1)
      IF(RANG.NE.0.0) DET=RANG/2.D1
      IF(RANG.EQ.0.0) DET=1.D0
      IF(RANGQ.NE.0.0) DETQ=RANGQ/6.D0
      IF(RANGQ.EQ.0.0) DETQ=1.D0
C  INITIALIZE NONDIMENSIONAL DEFLECTION COEFFICIENTS B(N)
      B(1)=0.4D0
      DO 12 I=1,7
      B(I+1)=(-1.D0)**I*B(I)*0.1D0
   12 CONTINUE
C  STORE SET OF B(N) VALUES AS F(N)
      DO 13 I=1,8
      F(I)=B(I)
   13 CONTINUE
      WRITE(KO,250)
C  INITIALIZE LOADING CONSTANTS
      QSTAR=0.D0
C  VARY QSTAR
```

```
      DO 93 IQ=1,4
      VARY=0.D0
      IF(NST.LT.0.0)NSTAR=VARY
      IF(SST.LT.0.0)SSTAR=VARY
C   VARY NSTAR OR SSTAR
      DO 91 II=1,11
      IF(B(1).NE.0.0.AND.II.NE.1) GO TO 19
C   RESET INITIAL VALUES B(N)
      DO 17 I=1,8
      B(I)=F(I)
   17 CONTINUE
   19 CONTINUE
      IN=0
      D=1.1
      C=0.2D0
      GO TO 14
   15 CONTINUE
      IN=IN+1
      IF(IN.GE.20) GO TO 16
      D=C+D
      DO 14 I=1,8
      B(I)=F(I)/D
   14 CONTINUE
      IRITE=2
      R=0.4D0
      EPS=0.5D-12
C   USE MATHLIB SUBROUTINE TO SOLVE THE NONLINEAR EQUATIONS
      CALL ZEROIN(8,B,R,EPS,ICV,EVAL,IRITE)
      IF(ICV.GE.2) GO TO 15
      WRITE(KO,240)QSTAR,NSTAR,SSTAR,B,IN
      GO TO 92
   16 CONTINUE
      WRITE(KO,206)NN
      MOUSE=1
      GO TO 94
   92 CONTINUE
```

```
      SUM=0.0
C   STORE COMPUTED VALUES OF B(N) AS BB(N,NN)
      DO 18 N=1,8
      BB(N,NN)=B(N)
C   COMPUTE BENDING MOMENT AT MIDDLE OF SUPPORTS
      YSTAR=YS*B(N)*N**2*(-1)**N
      SUM=SUM+YSTAR
   18 CONTINUE
      NN=NN+1
      MYSTAR(II,IQ)=SUM
C   COMPUTE DEFLECTION AT CENTER OF PLATE
      W(II,IQ)=2.0*(B(1)+B(3)+B(5)+B(7))
C   COMPUTE EFFECTIVE WIDTH
      IF(NSTAR.EQ.0.0) GO TO 93
      CALL CPQ(0,B,C1,C0)
      AE(II-1,IQ)=NSTAR/(NSTAR-AEE*C1)
      WRITE(KO,230)W(II,IQ),MYSTAR(II,IQ),AE(II-1,IQ)
      GO TO 94
   93 CONTINUE
      WRITE(KO,230)W(II,IQ),MYSTAR(II,IQ)
   94 CONTINUE
      NS(II)=VARY*DET
      VARY=VARY+2.D0
      IF(NST.LT.0.0)NSTAR=VARY*DET
      IF(SST.LT.0.0)SSTAR=VARY*DET
   91 CONTINUE
      QS(IQ)=QSTAR
      QSTAR=QSTAR+2.D0*DETQ
   90 CONTINUE
C   NO PLOT IF A SOLUTION DID NOT CONVERGE
      IF(MOUSE.EQ.1) GO TO 50
C   OUTPUT DATA
      WRITE(KO,200)
      WRITE(KO,201)RHO,GAMMA,ETA
      IF(NST.LT.0.0)WRITE(KO,202)SSTAR
      IF(SST.LT.0.0)WRITE(KO,222)NSTAR
```

```
    IF(NST.LT.0.0)WRITE(KO,203)
    IF(SST.LT.0.0)WRITE(KO,233)
    NN=1
    DO 30  IQ=1,4
    WRITE(KO,204)QS(IQ)
    DO 31  II=1,11
    WRITE(KO,205)NS(II),(BB(N,NN),N=1,8)
    NN=NN+1
  31 CONTINUE
  30 CONTINUE
    GO TO 50
C  NUMBER OF VALUES OF EFFECTIVE WIDTH TO BE PLOTTED
    MM=2
    DO 89  I=1,9
    IF(AE(I,1).EQ.1.0.AND.AE(I+1,1).NE.1.0) MM=I
  89 CONTINUE
    NO=11-MM
C  ARRANGE DATA TO BE PLOTTED INTO VECTORS
    DO 40  I=1,11
    WO1(I)=W(I,1)
    WO2(I)=W(I,2)
    WO3(I)=W(I,3)
    WO4(I)=W(I,4)
    MY1(I)=MYSTAR(I,1)
    MY2(I)=MYSTAR(I,2)
    MY3(I)=MYSTAR(I,3)
    MY4(I)=MYSTAR(I,4)
  40 CONTINUE
    IF(SST.LT.0.0) GO TO 41
    DO 41  I=1,NO
    PS(I)=NS(I+MM)
    AE1(I)=AE(I+MM-1,1)
    AE2(I)=AE(I+MM-1,2)
    AE3(I)=AE(I+MM-1,3)
    AE4(I)=AE(I+MM-1,4)
  41 CONTINUE
```

```
C   PLOT NONDIMENSIONAL PLATE DEFLECTIONS
      CALL PICTUR(4.,4.,XAX,8,YAX1,4,NS,WO1,-11,SI,KO,NS,WO2,-11,SI,K2,
     1 NS,WO3,-11,SI,K4,NS,WO4,-11,SI,K6)
      IF(SST.LT.0.0) GO TO 42
C   PLOT NONDIMENSIONAL EFFECTIVE WIDTH
      CALL PICTUR(4.,4.,XAX,8,YAX2,4,PS,AE1,-NO,SI,KO,PS,AE2,-NO,SI,K2,
     1 PS,AE3,-NO,SI,K4,PS,AE4,-NO,SI,K6)
   42 CONTINUE
C   PLOT NONDIMENSIONAL BENDING MOMENT IN THE Y-DIRECTION
      CALL PICTUR(4.,4.,XAX,8,YAX3,8,NS,MY1,-11,SI,KO,NS,MY2,-11,SI,K2,
     1 NS,MY3,-11,SI,K4,NS,MY4,-11,SI,K6)
      GO TO 50
   51 CONTINUE
      CALL ENDPLT
      WRITE(KO,207)
  100 FORMAT(8F10.3)
  200 FORMAT('1'//,28X,'NONDIMENSIONAL DEFLECTION COEFFICIENTS OF AN',
     1 ' ORTHOTROPIC PLATE'//)
  201 FORMAT(38X,'RHO =',F6.3,2X,' GAMMA =',F6.3,2X,' ETA =',F6.3,2X/)
  202 FORMAT(52X,'SSTAR =',F6.2/)
  222 FORMAT(52X,'NSTAR =',F6.2/)
  203 FORMAT(5X,'NYSTAR',9X,'B11',10X,'B12',10X,'B13',10X,'B14',
     1 10X,'B15',10X,'B16',10X,'B17',10X,'B18'/)
  233 FORMAT(5X,'SSTAR',10X,'B11',10X,'B12',10X,'B13',10X,'B14',
     1 10X,'B15',10X,'B16',10X,'B17',10X,'B18'/)
  204 FORMAT(3X,'QSTAR =',F5.1)
  205 FORMAT(4X,F6.2,5X,8(F10.6,3X))
  206 FORMAT(5X,'SOLUTION DID NOT CONVERGE AT STAGE',I3/10X,'NO PLOT')
  207 FORMAT('1','TERMINATION OF RUN')
  230 FORMAT(3F10.5)
  240 FORMAT(11F10.5,I4)
  250 FORMAT('1',2X,'QSTAR',5X,'NSTAR',5X,'SSTAR',7X,'B11',7X,'B12',7X,
     1 'B13',7X,'B14',7X,'B15',7X,'B16',7X,'B17',7X,'B18'/4X,'W/H',
     2 6X,'MYSTAR',5X,'AE/A'/)
      STOP
      END
```

MAIN PROGRAM 3 - BEHAVIOUR

```
C  CALCULATION OF NONDIMENSIONAL DEFLECTION COEFFICIENTS OF A LARGE ASPECT
C  RATIO ORTHOTROPIC PLATE UNDER COMBINED LOADINGS
C  B(N)     =  NONDIMENSIONAL DEFLECTION COEFFICIENTS
C  RHO      =  VIRTUAL ASPECT RATIO OF ORTHOTRCPIC PLATE
C  ETA      =  TORSION COEFFICIENT OF ORTHOTROPIC PLATE
C  GAMMA    =  ORTHOTROPIC PLATE COEFFICIENT
C  QSTAR    =  NONDIMENSIONAL HYDROSTATIC PRESSURE LOADING
C  NSTAR    =  NONDIMENSICNAL AVERAGE INPLANE COMPRESSIVE LOADING
C  SSTAR    =  NONDIMENSIONAL INPLANE EDGE SHEAR LOADING (ALL EDGES)
C  CPQ      =  COEFFICIENT, QUADRATIC FUNCTIONS OF B(N)
C  APQ      =  COEFFICIENT
C  W/H      =  NONDIMENSIONAL DEFLECTION OF PLATE
C  SIGMA STAR  =  NONDIMENSIONAL PRINCIPAL STRESS
C  ITERATIVE SOLUTION OF EIGHT SIMULTANEOUS NONLINEAR CUBIC EQUATIONS
C  ( F(B)=0 ), USING THE MATHLIB SUBPROGRAM ZEROIN (AP-21).
C  SUBPROGRAM EVAL CALCULATES F(B) FOR A VECTOR OF B(N) VALUES.
C  SUBPROGRAM CPQ CALCULATES COEFFICIENTS CPQ OF USE IN THE
C  SIMULTANEOUS EQUATIONS.
      REAL*8 RHO,GAMMA,ETA,NSTAR,QSTAR,SSTAR
      REAL*8 R,EPS,D,C,B(8),V(8)
      REAL*4 NS(4),SS(4),QS(4)
      DIMENSION CS1(17,101),CS2(101),CS3(101),CS4(8,101),CT(2,17)
      DIMENSION T1(17),T2(17),T3(17),T4(17),T5(17),T6(8),T7(8),T8(8)
      DIMENSION T9(8),T10(8),F(101),BB(8,4),SUM(9)
      DIMENSION Y(5,4,101),Y1(101),Y2(101),Y3(101),Y4(101),Y5(101)
      DIMENSION XAX(1),XAX1(1),XAX2(1),YAX1(1),YAX2(3)
      COMMON RHO,GAMMA,ETA,NSTAR,QSTAR,SSTAR
      EXTERNAL EVAL
      DATA KI,KO/5,6/
      DATA XAX1/' Y/B'/,XAX2/' X/A'/
      DATA YAX1/' W/H'/,YAX2/' SIG','MA S','TAR '/
      DATA MM,S1,IS1/101,0.0,0/
      DATA PI/3.141593/
C  SUPPRESS ERRORS CAUSED BY ZERCIN SUBROUTINE IN PROCESS OF CONVERGING
C  TO A SOLUTION.
      CALL ERRSET (208,256,-1,1,1,209)
```

```
C   USE CALCOMP PLOTTER SUBROUTINES
      CALL NEWPLT('M10347','9782','VELLUM ','BLACK')
      NN=1
      A=12.0*(1.0-0.09)
      W=6.0/A
      XAX(1)=XAX1(1)
C   GENERATE MATRIX OF VALUES OF COSINE TERMS
      E=1.0/100.0
      FF=-0.5D0
      DO 70 I=1,MM
      F(I)=FF
      Q=0.0
      DO 71 IQ=1,17
      CS1(IQ,I)=COS(2.0*Q*PI*FF)
      Q=Q+1.0
   71 CONTINUE
      CS2(I)=COS(PI*FF)
      CS3(I)=CS1(2,I)
      DO 72 N=1,8
      CS4(N,I)=CS1(N+1,I)
   72 CONTINUE
      FF=FF+E
   70 CONTINUE
      V(1)=0.4D0
      DO 73 I=1,7
      V(I+1)=(-1.D0)**I*V(1)*0.1D0
   73 CONTINUE
   50 CONTINUE
      MOUSE=0
C   ENTER VIRTUAL ASPECT RATIO, ORTHOTROPIC MATERIAL CONSTANTS AND
C   LOADINGS.
      READ(KI,100)RHO,GAMMA,ETA,NSTAR,SSTAR,QSTAR,TB
      IZ=2
      IF(TB.NE.0.0) IZ=1
      IF(RHO.EQ.0.0) GO TO 51
      BETA=1.0/RHO
```

```
      IF(NN.EQ.1) WRITE(KO,208)RHO,GAMMA,ETA
      RH=RHO
      GA=GAMMA
      ET=ETA
      NS(NN)=NSTAR
      SS(NN)=SSTAR
      QS(NN)=QSTAR
      RHO4=RHO**4
      TN=-NSTAR/A
      TAUXY=-SSTAR*ETA*GAMMA/A
      TAUXY=TAUXY/(GAMMA*RHO**2*BETA)
      TAUXYS=TAUXY**2
      DO 61 N=1,8
      T5(N)=(-1.0)**(N+1)*W
      T6(N)=(1.0+4.0*0.03*(N/BETA)**2)*W
      T7(N)=BETA**2*0.03*(-1.0)**(N+1)*W
      T8(N)=(4.0*N**2+BETA**2*0.03)*W
      T9(N)=T5(N)+T6(N)
      T10(N)=T7(N)+T8(N)
   61 CONTINUE
C  INITIALIZE NONDIMENSIONAL DEFLECTION COEFFICIENTS B(N)
      DO 12 I=1,8
      B(I)=V(I)
   12 CONTINUE
      WRITE(KO,250)
      IN=0
      D=1.1
      C=0.2D0
      GO TO 14
   15 CONTINUE
      IN=IN+1
      IF(IN.GE.20) GO TO 16
      D=C+D
      DO 14 I=1,8
      B(I)=V(I)/D
   14 CONTINUE
```

```
      IRITE=2
      R=0.4D0
      EPS=0.5D-12
C   USE MATHLIB SUBROUTINE TO SOLVE THE NONLINEAR EQUATIONS
      CALL ZEROIN(8,B,R,EPS,ICV,EVAL,IRITE)
      IF(ICV.GE.2) GO TO 15
      WRITE(KO,240)QSTAR,NSTAR,SSTAR,B,IN
      GO TO 92
   16 CONTINUE
      WRITE(KO,206)NN
      MOUSE=1
   92 CONTINUE
C   STORE COMPUTED VALUES OF B(N) AS BB(N,NN)
      DO 18 N=1,8
      BB(N,NN)=B(N)
   18 CONTINUE
C   NO PLOT IF A SOLUTION DID NOT CONVERGE
      IF(MOUSE.EQ.1) GO TO 95
      M=0
      DO 40 IQ=1,17
      Q=M
      CALL CPQ(M,B,C1,C0)
      Q4=Q**4
      IF(M.EQ.0) CT(1,IQ)=0.0
      IF(M.NE.0) CT(1,IQ)=C0/(4.0*RHO4*Q4)
      CT(2,IQ)=C1/(4.0+8.0*GAMMA*(RHO*Q)**2+RHO4*Q4)
      M=M+1
   40 CONTINUE
C   GENERATE VECTORS OF THE TERMS
      Q=0.0
      DO 60 IQ=1,17
      G=RHO4*Q**2
      T2(IQ)=CT(1,IQ)*G
      T3(IQ)=CT(2,IQ)*G
      T1(IQ)=T2(IQ)+T3(IQ)
      T4(IQ)=CT(2,IQ)
```

```
      Q=Q+1.0
   60 CONTINUE
C  SUM UP TERMS TO CALCULATE TOTAL STRESSES AND DEFLECTION
      DO 80 I=1,MM
C  INITIALIZE SUMS
      DO 83 M=1,9
      SUM(M)=0.0
   83 CONTINUE
      DO 81 IQ=1,17
      SUM(1)=SUM(1)-T1(IQ)*CS1(IQ,I)
      SUM(2)=SUM(2)-T4(IQ)*CS1(IQ,I)
      SUM(3)=SUM(3)-(T2(IQ)+T3(IQ)*CS3(I))
      SUM(4)=SUM(4)-T4(IQ)*CS3(I)
   81 CONTINUE
      DO 82 N=1,8
      SUM(5)=SUM(5)-B(N)*(T5(N)+T6(N)*CS4(N,I))
      SUM(6)=SUM(6)-B(N)*(T7(N)+T8(N)*CS4(N,I))
      SUM(7)=SUM(7)-B(N)*T9(N)*CS2(I)
      SUM(8)=SUM(8)-B(N)*T10(N)*CS2(I)
      SUM(9)=SUM(9)+B(N)*(T5(N)/W+CS4(N,I))
   82 CONTINUE
      SIGX1=TN+SUM(1)+(-1.0)**IZ*SUM(5)
      SIGX1=SIGX1/(RHO4*BETA**2)
      SIGY1=SUM(2)+(-1.0)**IZ*SUM(6)
      SIGX2=TN+SUM(3)+(-1.0)**IZ*SUM(7)
      SIGX2=SIGX2/(RHO4*BETA**2)
      SIGY2=SUM(4)+(-1.0)**IZ*SUM(8)
      DEFL=SUM(9)
      Y(1,NN,I)=DEFL
C  PRINCIPAL STRESSES
      Z1=SQRT((SIGX1/2.0-SIGY1/2.0)**2+TAUXYS)
      Z2=SQRT((SIGX2/2.0-SIGY2/2.0)**2+TAUXYS)
C  FOR X=0 (ALONG Y-AXIS)
      Y(2,NN,I)=(SIGX1+SIGY1)/2.0-Z1
      Y(3,NN,I)=(SIGX1+SIGY1)/2.0+Z1
C  FOR Y=0 (ALONG X-AXIS)
```

```
      Y(4,NN,I)=(SIGX2+SIGY2)/2.0-Z2
      Y(5,NN,I)=(SIGX2+SIGY2)/2.0+Z2
      IF(I.EQ.51) WRITE(KO,301)
      IF(I.EQ.51) WRITE(KO,300)(Y(J,NN,I),J=1,3)
   80 CONTINUE
      NN=NN+1
      GO TO 50
   51 CONTINUE
C  **  PLOTS  **
      DO 90 J=1,5
      DO 91 I=1,MM
      Y1(I)=Y(J,1,I)
      Y2(I)=Y(J,2,I)
      Y3(I)=Y(J,3,I)
      Y4(I)=Y(J,4,I)
      Y5(I)=0.0
   91 CONTINUE
      IF(J.GT.3) XAX(1)=XAX2(1)
      IF(J.GT.1) GO TO 53
      CALL PICTUR(6.,4.,XAX,4,YAX1,4,F,Y1,MM,S1,IS1,F,Y2,MM,S1,IS1,
     1 F,Y3,MM,S1,IS1,F,Y4,MM,S1,IS1,F,Y5,MM,S1,IS1)
      GO TO 54
   53 CONTINUE
      CALL PICTUR(6.,4.,XAX,4,YAX2,12,F,Y1,MM,S1,IS1,F,Y2,MM,S1,IS1,
     1 F,Y3,MM,S1,IS1,F,Y4,MM,S1,IS1,F,Y5,MM,S1,IS1)
   54 CONTINUE
   90 CONTINUE
   95 CONTINUE
C  OUTPUT DATA
      WRITE(KO,200)
      WRITE(KO,201)RH,GA,ET
      NN=NN-1
      DO 31 II=1,NN
      WRITE(KO,202)NS(II),SS(II),QS(II)
      WRITE(KO,203)
      WRITE(KO,204)(BB(N,II),N=1,8)
```

```
 31 CONTINUE
    CALL ENDPLT
    WRITE(KO,207)
100 FORMAT(8F10.3)
200 FORMAT('1'//,28X,'NONDIMENSIONAL DEFLECTION COEFFICIENTS OF AN',
   1 ' ORTHOTROPIC PLATE'//)
201 FORMAT(38X,'RHO =',F6.3,2X,' GAMMA =',F6.3,2X,' ETA =',F6.3,2X///)
202 FORMAT(30X,'NSTAR =',F6.2,2X,' SSTAR =',F6.2,2X,' QSTAR =',F6.2/)
203 FORMAT(12X,'B11',10X,'B12',10X,'B13',10X,'B14',10X,'B15',10X,
   1 'B16',10X,'B17',10X,'B18'/)
204 FORMAT(8X,8(F10.6,3X)//)
206 FORMAT(5X,'SOLUTION DID NOT CONVERGE AT STAGE',I3/10X,'NO PLOT')
207 FORMAT('1','TERMINATION OF RUN')
208 FORMAT('0',38X,'RHO =',F6.3,2X,' GAMMA =',F6.3,2X,' ETA =',F6.3/)
240 FORMAT(11F10.5,I4)
250 FORMAT('0',2X,'QSTAR',5X,'NSTAR',5X,'SSTAR',7X,'B11',7X,'B12',7X,
   1 'B13',7X,'B14',7X,'B15',7X,'B16',7X,'B17',7X,'B18'/)
300 FORMAT(5X,3(F8.4,6X)//)
301 FORMAT(/5X,'DEFLECTION AND PRINCIPAL STRESSES AT THE CENTER ',
   1 'OF THE PLATE'//5X,'DEFLECTION',13X,'STRESSES'/)
    STOP
    END
```

SUBROUTINE EVAL 1

$(N_y^* = 0)$

```
      SUBROUTINE EVAL(B,U)
C  LOAD NSTAR IN X DIRECTION
      INTEGER S
      REAL*8 RHO,GAMMA,ETA,PI,RHC4,AP
      REAL*8 NSTAR,SSTAR,QSTAR
      REAL*8 A,C,Z,D,E,S2,ST2,SL,TC42P,DELNS,W,T7,V
      REAL*8 FN,FS,FNPS,FNMS,FM
      REAL*8 G0,G1,C0S,C1S,C0N,C1N,C0NPS,C1NPS,C0M,C1M
      DOUBLE PRECISION B(8),U(8),F(16),T(7),TC(4),TC4(4),APQ
      COMMON RHO,GAMMA,ETA,NSTAR,QSTAR,SSTAR
      APQ(P,Q)=AP/(P**4+2.D0*GAMMA*(RHO*P*Q)**2+(Q*RHO)**4)
      DATA G0,G1/0.D0,1.D0/
      PI=3.141592653589793
C  INITIALIZE ARRAY F FOR USE AS DOUBLE PRECISION INDICES
      F(1)=1.D0
      DO 11 I=1,15
      F(I+1)=F(I)+1.D0
   11 CONTINUE
C  CALCULATE REPETATIVE CONSTANTS USED IN SIMULTANEOUS EQUATIONS
      AP=3.D0/4.D0*(1.D0-9.D-2)
      RHO4=RHO**4
      A=2.D0*ETA*SSTAR*(4.D0/(PI*RHO))**2
      C=A/2.D0
      Z=2.D0*RHO**2*ETA
      D=-8.D0*QSTAR/PI
      E=-1.D0/RHO4
      V=2.D0*NSTAR/RHO4
C  GENERATE AND CALCULATE EQUATIONS
      DO 1 S=1,8
      T(1)=0.D0
      T(2)=0.D0
      T(3)=0.D0
      T(4)=0.D0
      T(7)=0.D0
      FS=F(S)
      S2=2.D0*FS**2
```

```
      ST2=(2.DO*FS)**2
      SL=(-1.DO)**(S+1)
      CALL CPO(S,B,C1S,COS)
      TC42P=(APQ(G1,FS)*C1S+2.DO*APQ(GO,FS)*COS)*S2
      DO 2 N=1,8
      FN=F(N)
      NPS=N+S
      FNPS=F(NPS)
      NMS=N-S
      FNMS=F(N)-F(S)
      M=IABS(NMS)
      IF(M.EQ.0) FM=0.DO
      IF(M.NE.0) FM=F(M)
C   FIRST TERM
      DELNS=0.DO
      IF(N.EQ.S)DELNS=1.DO+Z*ST2+RHO4*ST2**2
      TC(1)=B(N)*(((-1.DO)**NPS*2.DO)+DELNS)*E
      T(1)=T(1)+TC(1)
C   SECOND TERM
      TC(2)=0.DO
      DO 3 I=1,7,2
      IF(N.EQ.I) GO TO 4
      IF(I.GT.N) GO TO 5
    3 CONTINUF
      GO TO 5
    4 TC(2)=A*SL*B(N)
    5 T(2)=T(2)+TC(2)
C   THIRD TERM
      TC(3)=0.DO
      DO 6 I=1,15,2
      IF(NPS.EQ.I)GO TO 7
      IF(I.GT.NPS) GO TO 8
    6 CONTINUE
      GO TO 8
    7 TC(3)=C*FN* B(N)*(1.DO/FNPS+1.DO/FNMS)
    8 T(3)=T(3)+TC(3)
```

```
C   FOURTH TERM
      W=1.DO
      IF(N.EQ.S) W=2.DO
      CALL CPQ(N,B,C1N,CON)
      CALL CPQ(NPS,B,C1NPS,CONPS)
      CALL CPQ(M,B,C1M,COM)
      TC4(1)=2.DO*SL*(FN**2)*(APQ(G1,FN)*C1N+2.DO*APQ(GO,FN)*CON)
      TC4(2)=(-1.DO)**(N+1)*TC42P
      TC4(3)=0.DO
      IF (N.EQ.S) GO TO 14
      TC4(3)=(FS-FN)**2*(APQ(G1,FNPS)*C1NPS+2.DO*APQ(GO,FM)*COM)
   14 CONTINUE
      TC4(4)=FNPS**2*(2.DO*APQ(GO,FNPS)*CONPS+W*APQ(G1,FM)*C1M)
      T(4)=T(4)+B(N)*(TC4(1)+TC4(2)+TC4(3)+TC4(4))
C   SEVENTH TERM
      T7=V*(-1.DO)**NPS*B(N)
      T(7)=T(7)+T7
    2 CONTINUE
C   FIFTH AND SIXTH TERMS
      T(5)=(-1.DO)**(S+2)*D
      T(6)=NSTAR*B(S)/RHO4
      U(S)=T(1)+T(2)+T(3)+T(4)+T(5)+T(6)+T(7)
    1 CONTINUE
      RETURN
      END
```

SUBROUTINE EVAL 2

$(N_x^* = 0)$

```
      SUBROUTINE EVAL(B,U)
C   LOAD NSTAR IN Y DIRECTION
      INTEGER S
      REAL*8 RHO,GAMMA,ETA,PI,RHO4,AP
      REAL*8 NSTAR,SSTAR,QSTAR
      REAL*8 A,C,Z,D,E,S2,ST2,SL,TC42P,DELNS,W,T7,V
      REAL*8 FN,FS,FNPS,FNMS,FM
      REAL*8 G0,G1,C0S,C1S,C0N,C1N,C0NPS,C1NPS,C0M,C1M
      DOUBLE PRECISION B(8),U(8),F(16),T(7),TC(4),TC4(4),APQ
      COMMON RHO,GAMMA,ETA,NSTAR,QSTAR,SSTAR
      APQ(P,Q)=AP/(P**4+2.D0*GAMMA*(RHO*P*Q)**2+(Q*RHO)**4)
      DATA G0,G1/0.D0,1.D0/
      PI=3.141592653589793
C   INITIALIZE ARRAY F FOR USE AS DOUBLE PRECISION INDICES
      F(1)=1.D0
      DO 11 I=1,15
      F(I+1)=F(I)+1.D0
   11 CONTINUF
C   CALCULATE REPETATIVE CONSTANTS USED IN SIMULTANFOUS EQUATIONS
      AP=3.D0/4.D0*(1.D0-9.D-2)
      RHO4=RHO**4
      A=2.D0*(4.D0/PI)**2*ETA/RHO**2*SSTAR
      C=A/2.D0
      Z=2.D0*RHO**2*ETA
      D=-8.D0*QSTAR/PI
      E=-1.D0/RHO4
C   GENERATE AND CALCULATE EQUATIONS
      DO 1 S=1,8
      T(1)=0.D0
      T(2)=0.D0
      T(3)=0.D0
      T(4)=0.D0
      FS=F(S)
      S2=2.D0*FS**2
      ST2=(2.D0*FS)**2
      SL=(-1.D0)**(S+1)
```

```
      CALL CPQ(S,B,C1S,COS)
      TC42P=(APQ(G1,FS)*C1S+2.D0*APQ(G0,FS)*COS)*S2
      DO 2 N=1,8
      FN=F(N)
      NPS=N+S
      FNPS=F(NPS)
      NMS=N-S
      FNMS=F(N)-F(S)
      M=IABS(NMS)
      IF(M.EQ.0) FM=0.D0
      IF(M.NE.0) FM=F(M)
C   FIRST TERM
      DELNS=0.D0
      IF(N.EQ.S)DELNS=1.D0+Z*ST2+RHO4*ST2**2
      TC(1)=B(N)*(((-1.D0)**NPS*2.D0)+DELNS)*F
      T(1)=T(1)+TC(1)
C   SECOND TERM
      TC(2)=0.D0
      DO 3 I=1,7,2
      IF(N.EQ.I) GO TO 4
      IF(I.GT.N) GO TO 5
    3 CONTINUE
      GO TO 5
    4 TC(2)=A*SL*B(N)
    5 T(2)=T(2)+TC(2)
C   THIRD TERM
      TC(3)=0.D0
      DO 6 I=1,15,2
      IF(NPS.EQ.I)GO TO 7
      IF(I.GT.NPS) GO TO 8
    6 CONTINUE
      GO TO 8
    7 TC(3)=C*FN*B(N)*(1.D0/FNPS+1.D0/FNMS)
    8 T(3)=T(3)+TC(3)
C   FOURTH TERM
      W=1.D0
```

```
      IF(N.EQ.S) W=2.D0
      CALL CPQ(N,B,C1N,CON)
      CALL CPQ(NPS,B,C1NPS,CONPS)
      CALL CPQ(M,B,C1M,COM)
      TC4(1)=2.D0*SL*(FN**2)*(APQ(G1,FN)*C1N+2.D0*APQ(G0,FN)*CON)
      TC4(2)=(-1.D0)**(N+1)*TC42P
      TC4(3)=0.D0
      IF (N.EQ.S) GO TO 14
      TC4(3)=(FS-FN)**2*(APQ(G1,FNPS)*C1NPS+2.D0*APQ(G0,FM)*COM)
   14 CONTINUE
      TC4(4)=FNPS**2*(2.D0*APQ(G0,FNPS)*CONPS+W*APQ(G1,FM)*C1M)
      T(4)=T(4)+B(N)*(TC4(1)+TC4(2)+TC4(3)+TC4(4))
    2 CONTINUE
C   FIFTH AND SIXTH TERMS
      T(5)=(-1.D0)**(S+2)*D
      T(6)=NSTAR*B(S)*ST2
      U(S)=T(1)+T(2)+T(3)+T(4)+T(5)+T(6)
    1 CONTINUE
      RETURN
      END
```

SUBROUTINE ZEROIN

```
      SUBROUTINE ZERCIN(N,XO,RO,EPS,ICV,EVAL,IRITE)
C  CTC ORD PROGRAM NO. 9065
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSIONXO(20),X(20),F(20,20),H(20,20),Y(20),D(20),FO(20),FX(20)
      NUMB=0
      ICV=0
      MAX=10*N
      MAX=AMINO(MAX,50)
      CT=RO*RO
      N1= N-1
      CALL EVAL(XO,FO)
      NUMB=NUMB+1
      DO  1  I=1,N
    1 FX(I)=FO(I)
      GO TO 4
    2 DO 3  I=1,N
      FO(I)=FX(I)
    3 XO(I)=X(I)
C
C
C     USE I FOR THE ORTHOGONAL MATRIX.
C
C
    4 DO 5  I=1,N
      C= XO(I)
      XO(I)= XO(I)+RO
      CALL EVAL(XO, X)
      NUMB=NUMB+1
      XO(I)= C
      DO 5 J=1,N
    5 F(J,I)=  X(J) - FO(J)
      IT=1
      GO TO 18
    6 DO  7 I=1,N
    7 D(I)= -RO*FX(I)
    8 CON=0.
```

```
      RK= 0.0D0
      FSUM= 0.
      DO  9  I=1,N
      X(I)= D(I)+ XO(I)
      CON= CON + X(I)**2
      RK= RK + D(I)**2
    9 Y(I)= RK
      CALL EVAL(X,FX)
      NUMB=NUMB+1
     IF(CON.EQ.0.0) CON=1.0D-32
      CON= RK/CON
      ICV=ICV+1
      IF(IRITE.EQ.1)   WRITE(6,200) CON,( X(I),I=1,N)
      IF(IRITE.EQ.1)   WRITE(6,300)  NUMB,  (FX(I),I=1,N)
C
C
C     CHECK FOR CONVERGENCE AND NEARNESS TO THE SOLUTION.
C
C
      IF(ICV.GT.MAX)  GO TO 27
      IF(CON.LE. EPS)   GO TO 28
      IF(RK.GT.CT) GO TO 2
      DO  10  I=1,N
      F(I,N)= FO(I)-FX(I)
      H(I,N)=-D(I)
      FO(I)=FX(I)
      XO(I) = X(I)
      DO  10  J=1,N1
   10 H(I,J)=0.0
C
C
C     USE THE DIFFERENCE MATRIX AS THE ORTHOGONAL APPROXIMATION.
C
C
      DO 11 J=1,N1
      K= J+1
```

```
      VAL=Y(J)*Y(K)
      IF(VAL.FQ.0.0) VAL=1.0D-32
       CON=DSQRT(RK/ VAL          )
       H(K,J) = -Y(J)*CON
       DO 11 I=1,J
   11  H(I,J) = CON*D(K)*D(I)
       DO  14 J= 1,N1
       DO  12 I=1,N
   12  X(I)= XO(I) +H(I,J)
       CALL EVAL(X,FX)
       NUMB=NUMB+1
       DO 13 I=1,N
   13  F(I,J) = FX(I)-FO(I)
   14  CONTINUE
       DO  15  I=1,N
   15  FX(I)= FO(I)
       IT=2
       GO TO 18
   16  DO  17  I=1,N
       D(I)=0.
       DO  17  J=1,N
   17  D(I)=D(I)- H(I,J)*FX(J)
       GO TO 8
C
C
C      SOLVE THE SYSTEM F*A=FX AND STORE THE RESULT IN FX.  THE SYSTEM
C      IS SOLVED BY GAUSSIAN ELIMINATION WITH PARTIAL PIVOTIN
C
C
   18  DO  24  I=1,N1
       C=0.
       DO  19  J=I,N
       CON=DABS(F(J,I))
       IF(CON.LT.C)  GO TO 19
       C=CON
       INDX=J
```

```
   19  CONTINUE
       IF(INDX.EQ.I)  GO TO 21
       DO  20  J=I,N
       C=F(I,J)
       F(I,J)=F(INDX,J)
   20  F(INDX,J)=C
       C=FX(I)
       FX(I)=FX(INDX)
       FX(INDX)=C
   21  IF(F(I,I).EQ.0.0D0)  GO TO 24
       NXT=I+1
       DO  23  J=NXT,N
       IF(F(J,I).EQ.0.0D0)  GO TO 23
      IF(F(I,I).EQ.0.0) F(I,I)=1.0D-32
       C=F(J,I)/F(I,I)
       F(J,I)=0.
       DO  22  IP=NXT,N
   22  F(J,IP)=F(J,IP)   -   F(I,IP)*C
       FX(J)=FX(J)  -  FX(I)*C
   23  CONTINUE
   24  CONTINUE
      IF(F(N,N).EQ.0.0) F(N,N)=1.0D-32
       FX(N)=FX(N)/F(N,N)
       DO  26  I=1,N1
       NXT=N-I
       DO  25  J=NXT,N1
   25  FX(NXT)=FX(NXT)  -  F(NXT,J+1)*FX(J+1)
      IF(F(NXT,NXT).EQ.0.0) F(NXT,NXT)=1.0D-32
   26  FX(NXT)=FX(NXT)/F(NXT,NXT)
C
C
C      END GAUSSIAN PORTION AND RETURN TO MAIN PROGRAM.
C
C
       GO TO (6,16),IT
   27  ICV=2
```

```
      GO TO 29
 28   ICV=1
 29   DO  30  I=1,N
 30   FSUM= FSUM + FX(I)**2
      GO TO (31,32),ICV
 31   IF(FSUM.GT..5D-10)    ICV=0
 32   EPS=FSUM
      RO= CON
      DO  33  I=1,N
 33   XO(I)=X(I)
      IF(IRITE.EQ.1)  WRITE(6,400)  ICV
      IF(IRITE.EQ.1)   WRITE(6,200)  RO,(XO(I),I=1,N)
      IF(IRITE.EQ.0)   WRITE(6,300)  NUMB,  (FO(I),I=1,N)
      RETURN
200   FORMAT(3H X 1PD15.7,2X,7D15.7/(3H    17X,7D15.7))
300   FORMAT(3H * I6,11X,     7D15.7/(3H    17X,7D15.7))
400   FORMAT(4H *** I6)
      END
```

SUBROUTINE CPQ

```
      SUBROUTINE CPQ(I1,B,C1,C0)
      DOUBLE PRECISION B(8),X,Y,C1,C0
      N=I1+1
      GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17),N
1     X=B(1)**2+4.D0*B(2)**2+9.D0*B(3)**2+1.6D1*B(4)**2
     1 +2.5D1*B(5)**2+3.6D1*B(6)**2+4.9D1*B(7)**2+6.4D1*B(8)**2
      Y=-X
      GO TO 18
2     X=4.D0*B(1)*B(2)+1.2D1*B(2)*B(3)+2.4D1*B(3)*B(4)+4.0D1*B(4)*B(5)
     1 +6.0D1*B(5)*B(6)+8.4D1*B(6)*B(7)+1.12D2*B(7)*B(8)
      Y=2.D0*B(1)*(-B(1)-1.5D0*B(2)-B(3)+B(4)-B(5)+B(6)-B(7)+B(8))
     1 -B(3)*(1.3D1*B(2)+2.5D1*B(4))-B(5)*(4.1D1*B(4)+6.1D1*B(6))
     2 -B(7)*(8.5D1*B(6)+1.13D2*B(8))
      GO TO 18
3     X=-B(1)**2+6.D0*B(1)*B(3)+1.6D1*B(2)*B(4)+3.0D1*B(3)*B(5)
     1 +4.8D1*B(4)*B(6)+7.0D1*B(5)*B(7)+9.6D1*B(6)*B(8)
      Y=8.D0*B(2)*(-B(1)+B(2)-B(3)-1.5D0*B(4)-B(5)+B(6)-B(7)+B(8))
     1 -B(1)**2-1.0D1*B(1)*B(3)-3.4D1*B(3)*B(5)-5.2D1*B(4)*B(6)
     2 -7.4D1*B(5)*B(7)-1.D2*B(6)*B(8)
      GO TO 18
4     X=-4.D0*B(1)*B(2)+8.D0*B(1)*B(4)+2.D1*B(2)*B(5)+3.6D1*B(3)*B(6)
     1 +5.6D1*B(4)*B(7)+8.D1*B(5)*B(8)
      Y=1.8D1*B(3)*(-B(1)+B(2)-B(3)+B(4)-B(5)-1.5D0*B(6)-B(7)+B(8))
     1 -5.D0*B(1)*B(2)-1.7D1*B(1)*B(4)-2.9D1*B(2)*B(5)-6.5D1*B(4)*B(7)
     2 -8.9D1*B(5)*B(8)
      GO TO 18
5     X=-6.D0*B(1)*B(3)+1.D1*B(1)*B(5)-4.D0*B(2)**2+2.4D1*B(2)*B(6)
     1 +4.2D1*B(3)*B(7)+6.4D1*B(4)*B(8)
      Y=3.2D1*B(4)*(-B(1)+B(2)-B(3)+B(4)-B(5)+B(6)-B(7)-1.5D0*B(8))
     1 -1.D1*B(1)*B(3)-2.6D1*B(1)*B(5)-4.D0*B(2)**2-4.D1*B(2)*B(6)
     2 -5.8D1*B(3)*B(7)
      GO TO 18
6     X=-8.D0*B(1)*B(4)+1.2D1*B(1)*B(6)-1.2D1*B(2)*B(3)+2.8D1*B(2)*B(7)
     1 +4.8D1*B(3)*B(8)
      Y=5.D1*B(5)*(-B(1)+B(2)-B(3)+B(4)-B(5)+B(6)-B(7)+B(8))
     1 -1.7D1*B(1)*B(4)-3.7D1*B(1)*B(6)-1.3D1*B(2)*B(3)-5.3D1*B(2)*B(7)
```

```
     2 -7.3D1*B(3)*B(8)
       GO TO 18
 7     X=-1.D1*B(1)*B(5)+1.4D1*B(1)*B(7)-1.6D1*B(2)*B(4)+3.2D1*B(2)*B(8)
     1 -9.D0*B(3)**2
       Y=7.2D1*B(6)*(-B(1)+B(2)-B(3)+B(4)-B(5)+B(6)-B(7)+B(8))
     1 -2.6D1*B(1)*B(5)-5.D1*B(1)*B(7)-2.D1*B(2)*B(4)-6.8D1*B(2)*B(8)
     2 -9.D0*B(3)**2
       GO TO 18
 8     X=-1.2D1*B(1)*B(6)+1.6D1*B(1)*B(8)-2.D1*B(2)*B(5)-2.4D1*B(3)*B(4)
       Y=9.8D1*B(7)*(-B(1)+B(2)-B(3)+B(4)-B(5)+B(6)-B(7)+B(8))
     1 -3.7D1*B(1)*B(6)-6.5D1*B(1)*B(8)-2.9D1*B(2)*B(5)-2.5D1*B(3)*B(4)
       GO TO 18
 9     X=-1.4D1*B(1)*B(7)-2.4D1*B(2)*B(6)-3.D1*B(3)*B(5)-1.6D1*B(4)**2
       Y=1.28D2*B(8)*(-B(1)+B(2)-B(3)+B(4)-B(5)+B(6)-B(7)+B(8))
     1 -5.D1*B(1)*B(7)-4.D1*B(2)*B(6)-3.4D1*B(3)*B(5)-1.6D1*B(4)**2
       GO TO 18
10     X=-1.6D1*B(1)*B(8)-2.8D1*B(2)*B(7)-3.6D1*B(3)*B(6)-4.D1*B(4)*B(5)
       Y=-6.5D1*B(1)*B(8)-5.3D1*B(2)*B(7)-4.5D1*B(3)*B(6)-4.1D1*B(4)*B(5)
       GO TO 18
11     X=-3.2D1*B(2)*B(8)-4.2D1*B(3)*B(7)-4.8D1*B(4)*B(6)-2.5D1*B(5)**2
       Y=-6.8D1*B(2)*B(8)-5.8D1*B(3)*B(7)-5.2D1*B(4)*B(6)-2.5D1*B(5)**2
       GO TO 18
12     X=-4.8D1*B(3)*B(8)-5.6D1*B(4)*B(7)-6.0D1*B(5)*B(6)
       Y=-7.3D1*B(3)*B(8)-6.5D1*B(4)*B(7)-6.1D1*B(5)*B(6)
       GO TO 18
13     X=-6.4D1*B(4)*B(8)-7.0D1*B(5)*B(7)-3.6D1*B(6)**2
       Y=-8.0D1*B(4)*B(8)-7.4D1*B(5)*B(7)-3.6D1*B(6)**2
       GO TO 18
14     X=-8.0D1*B(5)*B(8)-8.4D1*B(6)*B(7)
       Y=-8.9D1*B(5)*B(8)-8.5D1*B(6)*B(7)
       GO TO 18
15     X=-9.6D1*B(6)*B(8)-4.9D1*B(7)**2
       Y=-1.D2*B(6)*B(8)-4.9D1*B(7)**2
       GO TO 18
16     X=-1.12D2*B(7)*B(8)
       Y=-1.13D2*B(7)*B(8)
```

```
      GO TO 18
17    X=-6.4D1*B(8)**2
      Y=X
   18 CONTINUE
      C1=Y-X
      C2=Y+X
      RETURN
      END
```

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